Comment on "Two-phase behavior of financial markets"

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In a recent article [1], the authors report evidence for an intriguing two-phase behavior of financial markets when studying the distribution of volume imbalance (Ω) conditional to the local intensity of its fluctuations (Σ). We show here that this apparent phase transition is a generic consequence of the conditioning and exists even in the absence of any non trivial collective phenomenon.

More precisely, to each trade i one can associate the volume of the trade q_i and the 'sign' of the trade a_i, with $a_i = +1$ for buyer initiated trades, and $a_i = -1$ for seller initiated trades. Then, the volume imbalance is defined as:

$$\Omega_t = \sum_{i=1}^{N_t} a_i q_i \equiv N_t \langle a_i q_i \rangle,$$

where N_t is the number of trades taking place in a small time interval Δt around t. The intensity of fluctuations, on the other hand, is defined as:

$$\Sigma_t = \langle |a_i q_i - \langle a_i q_i \rangle| \rangle.$$

The finding reported in [1] is that conditioned a small value of $\Sigma < \Sigma_c$, the probability distribution of Ω is unimodal, and maximum at zero, whereas for $\Sigma > \Sigma_c$, the distribution becomes bimodal.

Our main point is the following: take a symmetric random variable X, and an estimate A of its magnitude $|X|$. If we know that A is small, then X is probably around zero, whereas when A is known to be large there are two most probable values for X, namely large and positive or large and negative. This generic behavior is more precisely illustrated on a simple example. If Ω is an a priori Gaussian variable of zero mean and unit variance and $\Sigma = \beta\Omega^2 + \eta$ a noisy estimate of its magnitude (η is a Gaussian noise, independent from Ω, of variance σ^2, and $\beta > 0$) then:

$$P(\Omega|\Sigma) = \frac{1}{Z(\Sigma)} \exp\left(-\frac{\Omega^2}{2} - \frac{(\beta\Omega^2 - \Sigma)^2}{2\sigma^2}\right),$$

in which we recognize the standard 'Mexican hat' potential with a single maximum at zero if $\Sigma < \Sigma_c = \sigma^2/2\beta$ and two symmetric maxima $\pm\Omega^*$ otherwise, with $\Omega^* \sim (\Sigma - \Sigma_c)^{1/2}$. Note that Σ_c is finite as soon as β is non zero, i.e. if there are non zero correlations between Ω^2 and Σ. This effect is more general and does not require the above normality assumptions.

Now the local noise intensity Σ, as defined in Eq. [1], is in fact highly correlated with the magnitude of the volume imbalance Ω and therefore the observed 'phase transition' is most plausibly due to the above mechanism. We have indeed computed the covariance between Σ and Ω^2 assuming that each trade is independently buyer or seller initiated with probability one half. We also neglect the fluctuations of $N_t = N$. To make the computation more tractable without changing the conclusion, we define Σ' as $\langle (q_i a_i - \langle q_i a_i \rangle)^2 \rangle$, rather than with an absolute value. We find

$$\langle \Sigma' \, \Omega^2 \rangle_c = (N-1)(\langle q^4 \rangle - 3\langle q^2 \rangle^2) + \left(1 - \frac{3}{N}\right) \sum_{i \neq j = 1}^{N} \langle q_i^2 q_j^2 \rangle_c,$$

where $\langle . \rangle_c$ means connected averages. As soon as traded volumes have fat tails ($\langle q^4 \rangle > 3\langle q^2 \rangle^2$) and/or are positively correlated ($\langle q_i^2 q_j^2 \rangle_c > 0$) (see e.g. [2]), the last equation shows that Σ' and Ω^2 are indeed positively correlated (i.e. $\beta > 0$). Moreover, if volume correlations are long ranged, the correlation coefficient does not vanish even for N large.

The above mechanism generically leads to $\Omega^* \sim \sqrt{\Sigma - \Sigma_c}$ when $\log(P(\Omega))$ is smooth at the origin. However choosing $\log(P(\Omega)) \propto -|\Omega|$, a realistic choice for order imbalance, leads to $\Omega^* \sim \Sigma - \Sigma_c$ as observed in [1]. We therefore suggest that the 'two phase' behaviour of $P(\Omega|\Sigma)$ is a direct consequence of known statistical properties of traded volumes.

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References

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