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More stylized facts of financial markets: leverage effect and downside correlations

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Abstract

We discuss two more universal features of stock markets: the so-called leverage effect (a negative correlation between past returns and future volatility), and the increased downside correlations. For individual stocks, the leverage correlation can be rationalized in terms of a new 'retarded' model which interpolates between a purely additive and a purely multiplicative stochastic process. For stock indices a specific market panic phenomenon seems to be necessary to account for the observed amplitude of the effect. As for the increase of correlations in highly volatile periods, we investigate how much of this effect can be explained within a simple non-Gaussian one-factor description with *time independent* correlations. In particular, this one-factor model can explain the level and asymmetry of empirical exceedance correlations, which reflects the fat-tailed and negatively skewed distribution of market returns. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The construction of adequate agent based models of financial markets is becoming a very active field of research. The hope is to understand in details collective effects in a human activity that is particularly well documented: high frequencies time series of thousands of different financial assets are now available for sophisticated statistical investigations. An important task of these investigations is to unveil several robust

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'stylized fact' of financial markets, which consistently appear on different markets and in different periods of time, and that any candidate model should convincingly explain. Some of these stylized facts have been known for a long time, such as the absence of significant linear correlation of returns (except on short time scales) or the 'fat tails' in the distribution of returns. More recent results have established the long ranged nature of volatility [1–9], and volume correlations [10] and the related 'multiscaling' behaviour observed for higher order correlation functions [9,11]. The aim of this paper is to report on two more important 'stylized facts' that seem to be rather universal: (a) the so-called 'leverage' effect (the volatility of stocks tends to increase when the price drops) [12] and (b) the apparent increase of cross correlations in highly volatility market conditions, in particular when prices significantly. We present some quantitative empirical evidence for these effects, provide simple models for their interpretation and discuss their connections. These effects are particularly important for option markets ([7,13,14]) and for risk management.

2. The leverage effect

2.1. Empirical results

We will call $S_i(t)$ the price of stock i at time t, and $\delta S_i(t)$ the (absolute) daily price change: $\delta S_i(t) = S_i(t+1) - S_i(t)$. The relative price change will be denoted as $\delta x_i(t) = \delta S_i(t)/S_i(t)$. Taking δx^2 as a proxy for the squared volatility, the leverage correlation function can be defined as

$$\mathcal{L}_i(\tau) = \frac{1}{Z}\langle[\delta x_i(t+\tau)]^2 \delta x_i(t)\rangle \,. \tag{1}$$

The coefficient Z is a normalization that we have chosen to be $Z = \langle \delta x_i(t)^2\rangle^2$ for reasons that will become clear below. (Note that since δx is dimensionless, $\mathcal{L}_i(\tau)$ is also dimensionless, despite this apparent lack of homogeneity between the numerator and denominator.)

In the following, we will consider $\tau \geqslant 1$. Negative values of τ lead to very small values of the correlation function, indistinguishable from noise. In other words, the correlation exists between future volatilities and past price changes; conversely, volatility changes do not convey any useful information on future price changes.

We have analyzed a set of 437 US stocks, extracted from the S&P 500 index and a set of 7 major international stock indices. Our dataset consisted of daily data ranging from January 1990 to May 2000 for stocks and from January 1990 to October 2000 for indices. We have computed $\mathcal{L}_i$ both for individual stocks and stock indices. The raw results are rather noisy. We have therefore assumed that individual stocks behave similarly and averaged $\mathcal{L}_i$ over the 437 different stocks in our dataset to give $\overline{\mathcal{L}_S}$, and over 7 different indices (from the US, Europe and Asia), to give $\overline{\mathcal{L}_I}$. The results are given in Figs. 1 and 2, respectively. As can be seen from these figures, both $\overline{\mathcal{L}_S}$ and

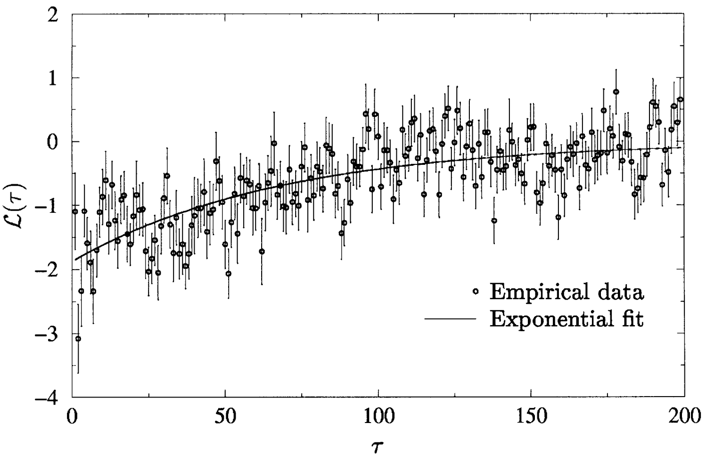

Fig. 1. Return-volatility correlation for individual stocks. Data points are the empirical correlation averaged over 437 US stocks, the error bars are two sigma errors bars estimated from the inter-stock variability. The full line shows an exponential fit (Eq. (2)) with $A_S = 1.9$ and $T_S = 69$ days. Note that $\mathscr{L}(\tau = 0)$ is not far from -2, as our retarded model predicts.

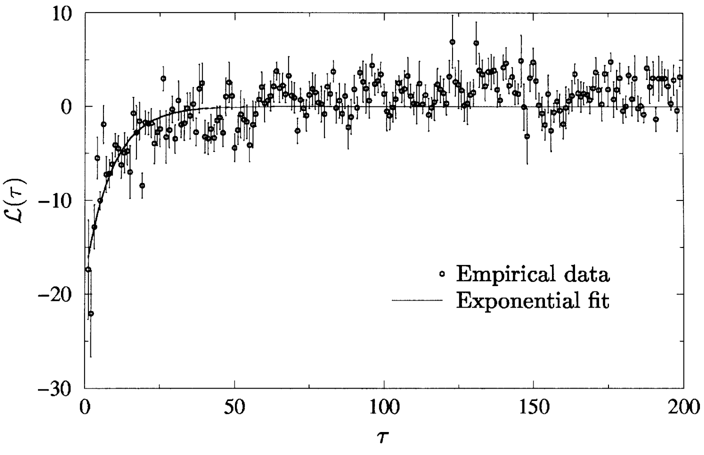

Fig. 2. Return-volatility correlation for stock indices. Data points are the empirical correlation averaged over 7 major stock indices, the error bars are two sigma errors bars estimated from the inter-index variability. The full line shows an exponential fit (Eq. (2)) with $A_I = 18$ and $T_I = 9.3$ days.

$\overline{\mathscr{L}_I}$ are clearly negative: price drops increase the volatility—this is the leverage effect. These correlation functions can rather well be fitted by *single* exponentials:

$$\overline{\mathscr{L}_{S,I}}(\tau) = -A_{S,I} \exp\left(-\frac{\tau}{T_{S,I}}\right) . \tag{2}$$

For US stocks, we find $A_S = 1.9$ and $T_S \simeq 69$ days (although this time scale does itself depend on the analyzed period), whereas for indices the amplitude A_I is significantly larger, $A_I = 18$ and the decay time shorter: $T_I \simeq 9$ days. This exponential decay should be contrasted with the very slow, power-law like decay of the volatility correlation function, which cannot be characterized by a unique decay time [1,3–9]. Therefore, a new time scale seems to be present in financial markets, intermediate between the very high frequency time scale seen on the correlation function of returns (several minutes) and the very low frequency time scales appearing in the volatility correlation function.

2.2. A retarded volatility model

Traditional models of asset fluctuations postulate that price changes are proportional to prices themselves. The price increment is therefore written as:

$$\delta S_i(t) = S_i(t)\,\sigma_i \varepsilon(t)\,, \tag{3}$$

where σ_i is the volatility and ε a random variable with zero mean and unit variance, independent of all past history. Eq. (3) means that price increments are at any time proportional to the current value of the price. Although it is true that on the long run, price increments tend to be proportional to prices themselves, this is not reasonable on short time scales. Locally, prices evolve in discrete steps (ticks), following buy or sell orders that can only be expressed an integer number of contracts. The mechanisms leading to price changes are therefore not expected to vary continuously as prices evolve, but rather to adapt only progressively if prices are seen to rise (or decrease) significantly over a certain time window. The model we propose to describe this lagged response to price changes is to replace S_i in Eq. (3) by a moving average S_i^R over a certain time window. More precisely, we write:

$$\delta S_i(t) = S_i^R(t)\,\sigma_i \varepsilon, \quad S_i^R(t) = \sum_{\tau=0}^{\infty} \mathcal{K}(\tau) S_i(t-\tau)\,, \tag{4}$$

where $\mathcal{K}(\tau)$ is a certain averaging kernel, normalized to one:

$$\sum_{\tau=0}^{\infty} \mathcal{K}(\tau) \equiv 1\,. \tag{5}$$

For example, an exponential moving average corresponds to $\mathcal{K}(\tau) = (1-\alpha)\alpha^\tau$, $(\alpha < 1)$. It is more convenient to rewrite S_i^R as

$$\begin{aligned} S_i^R(t) &= \sum_{\tau=0}^{\infty} \mathcal{K}(\tau) \left[S_i(t) - \sum_{\tau'=1}^{\tau} \delta S_i(t-\tau') \right] \\ &= S_i(t) - \sum_{\tau'=1}^{\infty} \bar{\mathcal{K}}(\tau') \delta S_i(t-\tau')\,, \end{aligned} \tag{6}$$

where $\bar{\mathcal{K}}(\tau)$ is the (discrete) integral of $\mathcal{K}(\tau)$:

$$\bar{\mathcal{K}}(\tau) = \sum_{\tau'=\tau}^{\infty} \mathcal{K}(\tau')\,. \tag{7}$$

Note that from the normalization of $\mathcal{K}(\tau)$, one has $\bar{\mathcal{K}}(0)=1$, independently of the specific shape of $\mathcal{K}(\tau)$. This is a crucial point in the following discussion.

For the exponential model, one finds $\bar{\mathcal{K}}(\tau)=\alpha^\tau$. The limit $\alpha \to 1$ corresponds to the case where $S_i^R(t)$ is a constant, and therefore Eq. (4) corresponds to an *additive* model. The other limit $\alpha \to 0$ (infinitely small averaging time window) leads to $S_i^R(t) \equiv S_i(t)$ and corresponds to a purely *multiplicative* model. A value of α close to one, $\alpha=1-\varepsilon$ corresponds to an additive model on short time scales ($\ll T=\varepsilon^{-1}$) and to a multiplicative model for long time scales ($\gg T$) [7].

One can easily calculate the correlation function $\mathcal{L}_i(\tau)$ to first order in the fluctuations. One finds [15]:

$$\mathcal{L}_i(\tau)=-2\bar{\mathcal{K}}(\tau)\,. \tag{8}$$

A very important prediction of this model is that $\mathcal{L}_i(\tau \to 0)=-2$. As shown in Fig. 1, this is indeed rather well obeyed for individual stocks, with $\bar{\mathcal{K}}(\tau)$ given by a simple exponential. We have confirmed this finding by analyzing a set of 500 European stocks and 300 Japanese stocks, again in the period 1990–2000. We find an exponential behavior with a time scale on the order of 40 days and, more importantly, and initial values of $\mathcal{L}_i$ close to the retarded model value -2. A similar result was obtained for Japanese stocks as well, which is interesting since this market did behave very differently both from the U.S. and European markets during the investigated time period. For the Japanese market, the prediction $\mathcal{L}(\tau \to 0)=-2$ not as good: an exponential fit of $\mathcal{L}(\tau)$ gives and $A_S=1.5$ and $T_S=47$ days.

We conclude that the leverage effect for stocks might not have a very deep economical significance, but can be assigned to a simple 'retarded' effect, where the change of prices are calibrated not on the instantaneous value of the price but on an exponential moving average of the price. On the other hand, as Fig. 2 reveals, the correlation function for indices has a much larger value for $\tau=0$ and the above interpretation cannot hold.

2.3. Leverage effect for indices

Fig. 2 also shows that the 'leverage effect' for indices tends to decay to zero much faster with the lag τ. This is at first sight puzzling, since the stock index is, by definition, an average over stocks. So one can wonder why the strong leverage effect for the index does not show up in stocks and conversely why the long time scale seen in stocks disappears in the index. One can actually show (see [15]) that these seemingly paradoxical features can be rationalized by a simple one factor model, where the 'market factor' is responsible for the strong leverage effect [15]. The one factor model assumes that the stock price increments can be written as

$$\delta S_i(t)=S_i^R(t)[\beta_i\phi(t)+\varepsilon_i(t)]\,, \tag{9}$$

where $\phi(t)$ is the return factor common to all the stocks, β_i are some time independent coefficients, and ε_i are the so-called idiosyncrasies, uncorrelated from stock to stock and from the common factor ϕ. The empirical leverage effect on indices shown in Fig. 2 suggests that there exists an index specific 'panic-like' effect, resulting from an increase of activity when the market goes down as a whole. A natural question is why this panic effect does not show up also in the statistics of individual stocks. A detailed analysis of the one factor model actually indicates that, to a good approximation, the index specific leverage effect and the stock leverage effect do not mix in the limit where the volatility of individual stocks is much larger than the index volatility [15]. Since the stock volatility is indeed found to be somewhat larger than the index volatility, one expects the mixing to be small and hardly detectable.

3. Cross-correlations in highly volatile periods

It is a common belief that cross-correlations between stocks actually *fluctuate* in time, and increase substantially in a period of high market volatility. This has been discussed in many papers—see for example [16–18], with more recent discussions, including new indicators, in [19–22]. Furthermore, this increase is thought to be larger for large downward moves than for large upward moves. The dynamics of these correlations themselves, and their asymmetry, should be estimated, leading to rather complex models [2,12,20,23].

A simpler point of view is provided by the (non-Gaussian) one factor model discussed above which encodes, by construction, a fixed correlation structure [24]. However, as we show below, this model is able to explain most of the apparent increase of correlations in volatile periods. Actually, as we show below, the time fluctuations of the measured cross-correlations between stocks is directly related to the fluctuations of the market volatility. This observation, together with the leverage effect discussed above, also explains why the increase appears to be stronger for downside moves—this is yet another manifestation of the negative skewness induced by the leverage effect.

3.1. Conditioning on absolute market return

We have first studied a measure of correlations between stocks conditioned on an extreme market return. A natural measure is given by the following coefficient:

$$\rho_>(\lambda) = \frac{\frac{1}{N^2}\sum_{i,j}(\langle r_i r_j\rangle_{>\lambda} - \langle r_i\rangle_{>\lambda}\langle r_j\rangle_{>\lambda})}{\frac{1}{N}\sum_i(\langle r_i^2\rangle_{>\lambda} - \langle r_i\rangle_{>\lambda}^2)}\,, \tag{10}$$

where the subscript $>\lambda$ indicates that the averaging is restricted to market returns r_m in absolute value larger than λ. For $\lambda = 0$ the conditioning disappears.

In a first approximation, the distribution of individual stocks returns can be taken to be symmetrical, leading $\langle r_i\rangle_{>\lambda} \simeq 0$. The above equation can therefore be

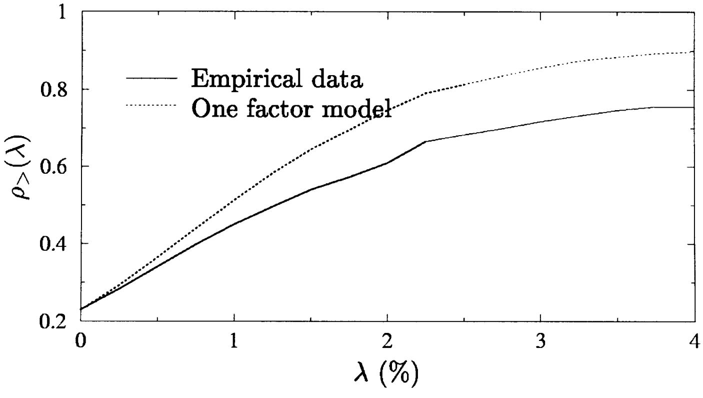

Fig. 3. Correlation measure $\rho_>(\lambda)$ conditional to the absolute market return to be larger than λ, both for the empirical data and the one-factor model. Note that both show a similar apparent increase of correlations with λ. This effect is actually overestimated by the one-factor model with fixed residual volatilities.

transformed into:

$$\rho_>(\lambda) \simeq \frac{\sigma_m^2(\lambda)}{\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2(\lambda)}, \tag{11}$$

where $\sigma_m^2(\lambda)$ is the market volatility conditioned to market returns in absolute value larger than λ, and $\sigma_i^2(\lambda) = \langle r_i^2\rangle_{>\lambda} - \langle r_i\rangle_{>\lambda}^2$. In the context of a one-factor model, we therefore obtain:

$$\rho_>(\lambda) = \frac{\sigma_m^2(\lambda)}{(\frac{1}{N}\sum_{i=1}^{N}\beta_i^2)\sigma_m^2(\lambda) + \frac{1}{N}\sum_{i=1}^{N}\sigma_{\varepsilon_i}^2}. \tag{12}$$

Within the simplest one factor model, the residual volatilities $\sigma_{\varepsilon_i}^2$ are independent of r_m and therefore of λ whereas $\sigma_m^2(\lambda)$ is obviously an increasing function of λ. Hence the coefficient $\rho_>(\lambda)$ is an increasing function of λ. The one-factor model therefore predicts an increase of the correlations (as measured by $\rho_>(\lambda)$) in high volatility periods. This conclusion is quite general, it holds in particular for any factor model, even with Gaussian statistics. More precisely, we can now compare the coefficient $\rho_>(\lambda)$ measured empirically to one obtained with surrogate data generated consistently with the one factor model. (The precise procedure to generate these surrogate data is presented in the Appendix.) The results are presented in Fig. 3. Interestingly, the surrogate and empirical correlations are similar, displaying qualitatively the same increase of the cross-correlation when conditioned to large market returns. This shows that a one-factor model does indeed account quantitatively for the apparent increase of cross-correlations in high volatility periods.

The one-factor model actually even *overestimates* the correlations for large λ. This fact can be understood qualitatively as a result of a positive correlation between the amplitude of the market return $|r_m|$ and the residual volatilities σ_{ε_i} (see e.g., [24]). For large values of λ, σ_{ε_i} is found to be larger than its average value. From Eq. (12), this lowers the correlation $\rho_>(\lambda)$ as compared to the simplest one-factor model where the volatility fluctuations of the residuals are neglected.

3.2. Growth and asymmetry of exceedance correlations

Another interesting quantity that has been much studied in the econometric literature recently, is the so-called exceedance correlation function, introduced in [19]. One first defines normalized centered returns $\tilde{r}_i$ with zero mean and unit variance. The positive exceedance correlation between i and j is defined as:

$$\rho_{ij}^{+}(\theta) = \frac{\langle \tilde{r}_i \tilde{r}_j \rangle_{>\theta} - \langle \tilde{r}_i \rangle_{>\theta} \langle \tilde{r}_j \rangle_{>\theta}}{\sqrt{(\langle \tilde{r}_i^2 \rangle_{>\theta} - \langle \tilde{r}_i \rangle_{>\theta}^2)(\langle \tilde{r}_j^2 \rangle_{>\theta} - \langle \tilde{r}_j \rangle_{>\theta}^2)}}\,, \tag{13}$$

where the subscript $> \theta$ means that both normalized returns are larger than θ. Large θ's correspond to extreme correlations. The negative exceedance correlation $\rho_{ij}^{-}(\theta)$ is defined similarly, the conditioning being now on returns smaller than θ. Fig. 4 shows the exceedance correlation function, averaged over the pairs i and j, both for real data and for the surrogate one-factor model data. As in previous papers, we have shown $\rho_{ij}^{+}(\theta)$ for positive θ and $\rho_{ij}^{-}(\theta)$ for negative θ. As in many other studies [19–22], we find that $\rho^{\pm}(\theta)$ *grows* with $|\theta|$ and is larger for large negative moves than for large positive moves. This is in strong contrast with the prediction of a Gaussian model, which gives a symmetric tent-shaped graph that goes to zero for large $|\theta|$. This effect seems to be rather universal, and has been observed in various different situations (stock markets, international indices, foreign exchange, etc.).

Several models have been considered to explain the observed results [20–22]. Simple GARCH or Jump models cannot account for the shape of the exceedance correlations. Qualitatively similar graphs can however be reproduced within a rather sophisticated 'regime switching' model, where the two assets switch between a positive, low volatility trend with small cross-correlations and a negative, high volatility trend with large

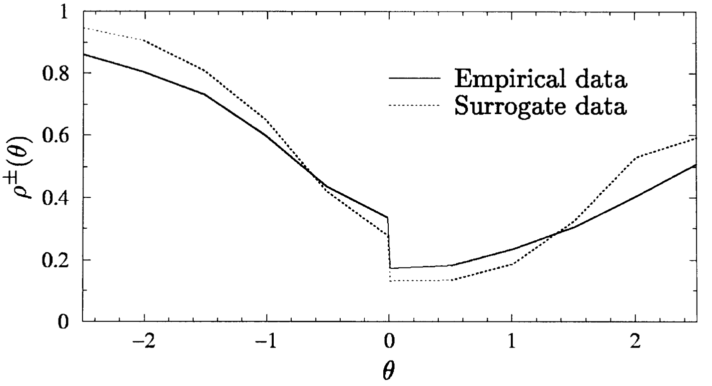

Fig. 4. Average exceedance correlation functions between stocks as a function of the level parameter θ, both for real data and the surrogate one-factor model. We have shown $\rho_{ij}^{+}(\theta)$ for positive θ and $\rho_{ij}^{-}(\theta)$ for negative θ. Note that this quantity *grows* with $|\theta|$ and is strongly asymmetric.

cross-correlations. Note that by construction, this ‘regime switching’ model induces a strong skew in the ‘index’ (i.e., the average between the two assets). Fig. 4 however clearly shows that a *fixed* correlation non-Gaussian one-factor model is enough to explain quantitatively the level and asymmetry of the exceedance correlation function. In particular, the asymmetry is induced by the large negative skewness in the distribution of index returns, and the growth of the exceedance correlation with $|\theta|$ is related to distribution tails fatter than exponential (in our case, these tails are indeed power-laws).

4. Conclusion

In this paper, we have reported on two more stylized facts of financial markets, the leverage effect, and the apparent increase of cross-correlations in highly volatile periods, in particular for downside moves.

We have found that the leverage correlation is moderate and decays over a few months for individual stocks, and much stronger but decaying much faster for stock indices. In the case of individual stocks, we have found that the magnitude of this correlation can be rationalized in terms of a retarded effect, which only assumes that the reference price used to set the scale for price updates is *not* the instantaneous price but rather a moving average of the price over the last few months. This interpretation, supported by the data on U.S., European and Japanese stocks, appears to us rather likely, and defines an interesting class of stochastic processes, intermediate between purely additive (valid on short time scales) and purely multiplicative (relevant for long time scales), first advocated in [7]. For stock indices, however, this interpretation breaks down and a specific market panic phenomenon seems to be responsible for the enhanced observed negative correlation between volatility and returns (and in turn to the strong skews observed on index option smiles).

We have also shown that the apparent increase of correlation between stock returns in extreme conditions can be satisfactorily explained within a *static* one-factor model which accounts for fat-tail effects. In this model, conditioning on a high observed volatility naturally leads to an increase of the apparent correlations. The much discussed exceedance correlations can also be reproduced quantitatively and reflects both the non-Gaussian nature of the fluctuations and the negative skewness of the index, and *not* the fact that correlations themselves are time dependent.

This one-factor model is however only an approximation to the true correlations, and more subtle effects (such as the ‘ensemble’ skewness recently discovered by Lillo and Mantegna [25]) require an extension of the one factor model, where the variance and skewness of the residuals themselves depend on the market return.

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Appendix A. One factor surrogate data

The procedure we used to generate the surrogate data is the following:

- Compute the β_i's using Eq. (9) (with $S_R = S$) over the whole time period. These β_i's are found to be rather narrowly distributed around 1, with $(1/N)\sum_{i=1}^{N} \beta_i^2 = 1.05$.
- Compute the variance of the residuals $\sigma_{\varepsilon_i}^2 = \sigma_i^2 - \beta_i^2 \sigma_m^2$. On the dataset we used σ_m was 0.91% (per day) whereas the rms σ_{ε_i} was 1.66%.
- Generate the residual $\varepsilon_i(t) = \sigma_{\varepsilon_i} u_i(t)$, where the $u_i(t)$ are independent random variables of unit variance with a leptokurtic (fat tailed) distribution—we have chosen here a Student distribution with an exponent $\mu = 4$:

$$P(u) = \frac{3}{(2 + u^2)^{5/2}}, \tag{A.1}$$

  which is known to represent adequately the empirical data (see, e.g. [26–28]).
- Compute the surrogate return as $r_i^{\text{surr}}(t) = \beta_i r_m(t) + \varepsilon_i(t)$, where $r_m(t)$ is the *true* market return at day t.

Within this method, both the empirical and surrogate returns are based on the very same realization of the market statistics. This allows us to compare meaningfully the results of the surrogate model with real data, without further averaging. It also short-cuts the precise parameterization of the distribution of market returns, in particular its correct negative skewness, which turns out to be crucial.

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