

Random walks, liquidity molasses and critical response in financial markets

Jean-Philippe Bouchaud*, Julien Kockelkoren*, Marc Potters*

August 17, 2004

* Science & Finance, Capital Fund Management, 6-8 Bvd Haussmann
75009 Paris, France

Abstract

Stock prices are observed to be random walks in time despite a strong, long term memory in the signs of trades (buys or sells). Lillo and Farmer have recently suggested that these correlations are compensated by opposite long ranged fluctuations in liquidity, with an otherwise *permanent* market impact, challenging the scenario proposed in Quantitative Finance 4, 176 (2004), where the impact is *transient*, with a power-law decay in time. The exponent of this decay is precisely tuned to a critical value, ensuring simultaneously that prices are diffusive on long time scales and that the response function is nearly constant. We provide new analysis of empirical data that confirm and make more precise our previous claims. We show that the power-law decay of the bare impact function comes both from an excess flow of limit order opposite to the market order flow, and to a systematic anti-correlation of the bid-ask motion between trades, two effects that create a ‘liquidity molasses’ which dampens market volatility.

1 Introduction

The volatility of financial assets is well known to be too much large compared to the prediction of Efficient Market Theory [1] and to exhibit intriguing statistical anomalies, such as intermittency and long range memory (for recent reviews, see [2, 3, 4, 5]). The availability of all trades and quotes on electronic markets makes it possible to analyze in details the intimate mechanisms leading to these anomalies. In a previous paper [6], we have proposed, based on empirical data, that the random walk nature of prices (i.e. the absence of return autocorrelations) is in fact highly non trivial and results from a fine-tuned competition between liquidity providers and liquidity takers. In order not to reveal their strategy, liquidity takers must decompose their orders in small trades that are diluted in time over a several hours to several days. This creates long range persistence in the ‘sign’ of the market orders (i.e. buy, $\varepsilon = +1$ or sell $\varepsilon = -1$) [7, 8, 6, 9]. This persistence should naively lead to a positive correlations of the returns and a super-diffusive behaviour of the price [6, 9]. However, liquidity providers act such as to create long range anti-persistence in price changes: liquidity providers make their profit on the bid-ask spread but lose money when the price makes large excursions, in which case they sell low and have to buy high (or vice versa) for inventory reasons. Both effects rather precisely compensate and lead to an overall diffusive behaviour (at least to a first approximation), such that (statistical) arbitrage opportunities are absent, as expected. We have shown in [6] that this picture allows one to understand the temporal structure of the market impact function (which measures how a given trade affects on average future prices), which was found to first increase, reach a maximum and finally decrease at large time, reflecting the mean-reversion action of liquidity providers.

The above picture was recently challenged by Lillo and Farmer [9]. Although they also find long memory (i.e., non summable power-law correlations) in the sign of market orders, they claim that the compensating mechanism that leads to uncorrelated returns is not the slow, mean-reverting influence of liquidity providers suggested in [6]. Rather, they argue that long range *liquidity fluctuations*, correlated with the order flow, act to suppress the otherwise permanent impact of market orders and make the price diffusive.

The aim of this paper is to explain in more details the differences and similarities between these conflicting pictures, and to present new data that support our original assertions [6]. While our previous paper mainly discussed on the case of France-Telecom, we also present a more systematic account of our main observables for a substantial set of stocks from the Paris Bourse. We also give a much more precise qualitative and quantitative description of the way liquidity providers manage, on average, to mean-revert the price by monitoring the flow of limit orders. We therefore argue that liquidity providers create a kind of ‘liquidity molasses’ that stabilises the volatility of financial markets, which is indeed the traditional role given to market makers.

2 The impact of trades on prices

2.1 Formulation of the problem

In the following, we will consider follow the dynamics of prices in trade time n (i.e. each distinct trade increases n by one unit) and define prices p_n as the midpoint just before the n^{th} trade: $p_n = (a_n + b_n)/2$, where a_n and b_n are, respectively, the ask price and the bid price corresponding to the last quote before the trade. We assume that the price can be written in general as:

$$p_n = \sum_{n' < n} \mathcal{G}(n, n' | \varepsilon_{n'}, V_{n'}, \mathcal{S}_{n'}) \quad (1)$$

where \mathcal{G} describes the impact at time n of a trade at time n' , of sign and volume $\varepsilon_{n'}$, $V_{n'}$, knowing that the order book at time n' is in a certain state $\mathcal{S}_{n'}$ (specified by the list of all prices and volumes of the limit orders). The assumption we made in [6] is that the impact function \mathcal{G} can be decomposed into an average, systematic part in the direction of the trade, plus fluctuations:

$$\mathcal{G}(n, n' | \varepsilon_{n'}, V_{n'}, \mathcal{S}_{n'}) \equiv \varepsilon_{n'} G(n, n' | V_{n'}) + \xi(n, n'), \quad (2)$$

where the function G was furthermore assumed to be time translation invariant¹ and factorisable as: $G(n, n' | V_{n'}) = f(V_{n'}) G_0(n - n')$. The last assumption is motivated by theoretical and empirical results [10, 16, 6], where $f(V)$ is found to be a power-law with a small exponent $f(V) \sim V^\beta$ [14, 15] or a logarithm $f(V) \sim \ln V$ [16, 6]. The noise term $\xi(n, n')$ is uncorrelated with the $\varepsilon_{n'}$ and has a variance $(n - n')D$. The final form of the model proposed in [6] therefore reads:

$$p_n = \sum_{n' < n} G_0(n - n') \varepsilon_{n'} \ln V_{n'} + \xi(n, n'). \quad (3)$$

The main finding of [6] is that the bare impact function $G_0(\ell)$ must decay with the time lag in order to compensate for the long range correlation in the ε , in other words that the impact of a single trade is transient rather than permanent. In their recent work, Lillo and Farmer [9] argue that it is rather the fluctuations in liquidity (encoded in the instantaneous shape of the order book $\mathcal{S}_{n'}$), that are crucial. Their model amounts to write p_n as:

$$p_n = \sum_{n' < n} \frac{\varepsilon_{n'} V_{n'}^\beta}{\lambda(\mathcal{S}_{n'})} + \xi(n, n'), \quad (4)$$

with $\beta = 0.3$ and where λ is the instantaneous liquidity of the market. The difference between V^β and $\ln V$ is actually not relevant; rather, the crucial difference between Eq. (3) and Eq. (4) is that the impact is *transient* in the former

¹This is probably only an approximation since time of the day, for example, should matter.

case and *permanent* (but fluctuating) in the latter case, a point on which we will comment later.

The argument of ref. [9] in favor of the second model, Eq. (4) goes in two steps: first, they propose, as a proxy of the instantaneous liquidity λ_n , the volume v_n at the best price (i.e. ask for buys and bid for sells): see [9] section VI B. They then study the time series of $r_n = \varepsilon_n V_n^\beta / v_n$ and find that linear correlations have nearly completely disappeared, at variance with the unrescaled series $\varepsilon_n V_n^\beta$ that exhibit the problematic long range correlations. Their conclusion is therefore that “the inclusion of the time varying liquidity term apparently removes long-memory”. Here, we want to refute this interpretation based on three independent sets of arguments: a) we show that Eq. (4) has less explicative power than Eq. (3); b) Eq. (4) leads to an average response function (see [6] and below) that significantly increases with time lag, at variance with data and c) the absence of linear correlations observed in r_n is an artefact coming from the very large fluctuations of the volume at the best price. Note that our data concerns stocks from Paris Bourse rather than the LSE stocks studied in [9]. However, we do not expect major qualitative differences between the two markets.

2.2 Response functions

We first start by recalling the definition of the average response function, as the correlation between the sign of a trade at time n and the subsequent price difference between n and $n + \ell$ [6]:

$$\mathcal{R}(\ell) = \langle (p_{n+\ell} - p_n) \cdot \varepsilon_n \rangle, \quad (5)$$

The quantity $\mathcal{R}(\ell)$ measures how much, on average, the price moves up conditioned to a buy order at time 0 (or how a sell order moves the price down) a time ℓ later. Note that because of the temporal correlations between the ε 's, this quantity is *not* the above market response to a single trade $G_0(\ell)$ [6]. This quantity is plotted in Fig. 1 for Carrefour in 2001, 2002. As emphasized in [16, 6], $\mathcal{R}(\ell)$ is found to weakly increase up to a maximum beyond which it decays back and can even change sign for large ℓ (see Figs. 2, 3). For other stocks, or other periods, the maximum of $\mathcal{R}(\ell)$ is not observed, and $\mathcal{R}(\ell)$ is seen to increase (although always rather mildly, at most by a factor 3) with ℓ : see Figs. 2,3. As will be clear below, this difference of behaviour can actually be understood within our model.

In Fig. 4, we also plot three other, similar quantities. The first is the (normalized) correlation between the price change and $\varepsilon_n \ln V_n$:

$$\mathcal{R}_V(\ell) = \frac{\langle (p_{n+\ell} - p_n) \cdot \varepsilon_n \ln V_n \rangle}{\langle \ln^2 V_n \rangle^{1/2}} \quad (6)$$

which has a similar shape but is distinctly larger than \mathcal{R} itself, showing that, as expected, the variable $\varepsilon_n \ln V_n$ has a larger explicative power than ε_n itself.

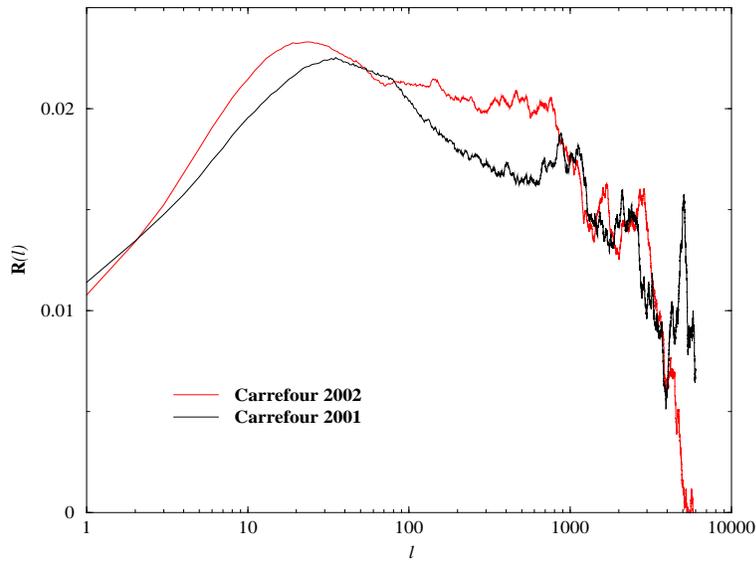


Figure 1: Response function $\mathcal{R}(\ell)$ (in Euros) for Carrefour in the periods 2001 and 2002.

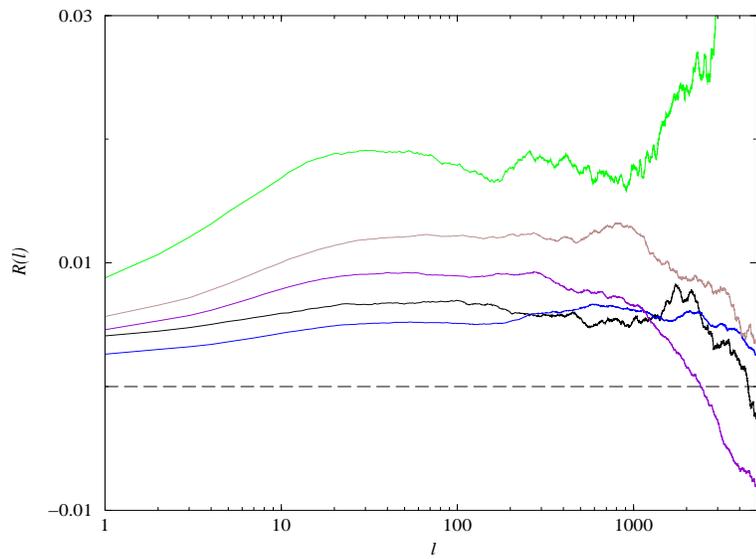


Figure 2: Response function $\mathcal{R}(\ell)$ (in Euros) for stocks from Paris Bourse in 2002. From top to bottom: EN, EX, FTE, ACA, CGE. (See Table 1 for the stocks code). Note that for some stocks $\mathcal{R}(\ell)$ increases for all ℓ (see e.g. CGE), whereas for other stocks $\mathcal{R}(\ell)$ reaches a maximum before becoming negative (see e.g. ACA). The dotted line correspond to $\mathcal{R}(\ell) = 0$.

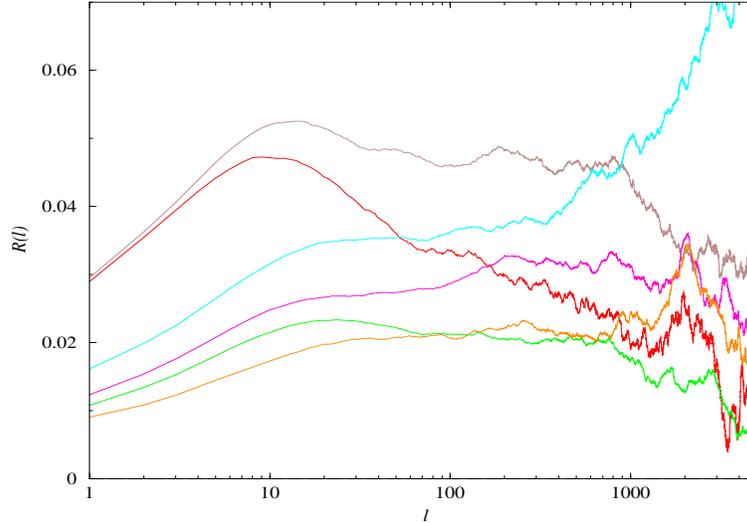


Figure 3: Response function $\mathcal{R}(\ell)$ (in Euros) for other stocks from Paris Bourse in 2002. From top to bottom: FP, BN, GLE, MC, CA, VIE. (See Table 1 for the stocks code). Note that for some stocks $\mathcal{R}(\ell)$ increases for all ℓ (see e.g. GLE), whereas for other stocks $\mathcal{R}(\ell)$ reaches a maximum before becoming negative (see e.g. CA, for $\ell > 5000$).

Code	Stock name	Av. price	Av. tick	Av. spread	# trades
ACA	Crédit Agricole	19.63	0.01	0.0408	379,000
BN	Danone	132.50	0.1	0.154	351,000
CA	Carrefour	48.54	0.0268	0.0578	555,000
CGE	Alcatel	9.85	0.01	0.015	1,020,000
EN	Bouygues	29.69	0.01	0.0413	240,000
EX	Vivendi	27.47	0.0126	0.0287	979,000
FP	Total	152.27	0.1	0.136	759,000
FTE	France-Telecom	21.04	0.01	0.024	1,051,000
GLE	Société Générale	61.80	0.043	0.0735	499,000
MC	LVMH	47.71	0.0209	0.0566	437,000
VIE	Vivendi Env.	29.75	0.01	0.0452	226,000

Table 1: Selection of stocks studied in this paper, with the average price, tick size and bid-ask spread in Euros in 2002. We also give the total number of trades in 2002. The results reported here qualitatively hold for most other stocks from Paris Bourse, but also other exchanges (see [6, 9]).

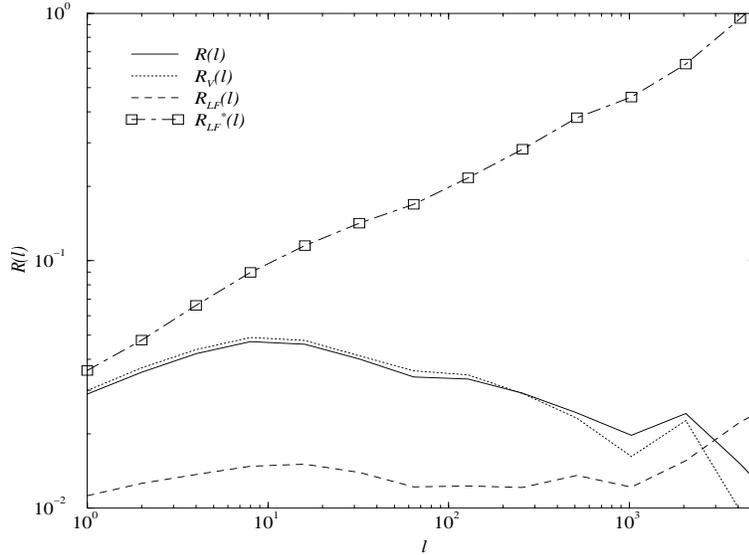


Figure 4: Four different ‘response functions’ $\mathcal{R}(\ell)$, $\mathcal{R}_V(\ell)$, $\mathcal{R}_{LF}(\ell)$ and $\mathcal{R}_{LF}^*(\ell)$, (see text) in Euros for BN in 2002. This plot shows (a) that the Lillo-Farmer variable r_n has a weak explicative power (see \mathcal{R}_{LF} – dashed line) and (b) that their permanent impact model leads to a considerable over-estimation of the true response function (see \mathcal{R}_{LF}^* – dashed-dotted lines, showing a 30 times increase with ℓ).

In order to test the Lillo-Farmer model, we have also computed two further quantities. One is the normalized correlation between the Lillo-Farmer variable $r_n = \varepsilon_n V_n^\beta / v_n$ and the empirical price change:

$$\mathcal{R}_{LF}(\ell) = \frac{\langle (p_{n+\ell} - p_n) \cdot r_n \rangle}{\langle r_n^2 \rangle^{1/2}}. \quad (7)$$

This quantity measures the explicative power of r_n , and can be directly compared to \mathcal{R} and \mathcal{R}_V . As can be seen in Fig. 4, $\mathcal{R}_{LF}(\ell)$ is in fact a factor 3 smaller than $\mathcal{R}_V(\ell)$ (see also the quantity Z in Table 2, last column).

The second interesting quantity is:

$$\mathcal{R}_{LF}^*(\ell) = \left\langle \left(\sum_{n'=n}^{n+\ell-1} r_{n'} \right) \cdot \varepsilon_n \right\rangle. \quad (8)$$

The quantity measures a fictitious average response function, which would follow if the price dynamics was given by Eq. (4). We see in Fig. 4 that $\mathcal{R}_{LF}^*(\ell)$, at variance with the true $\mathcal{R}(\ell)$, sharply grows with ℓ , as a consequence of the correlation of the ε 's which are not compensated by a fluctuating liquidity. As we have mentioned in [6], the response function $\mathcal{R}(\ell)$ is a very sensitive measure

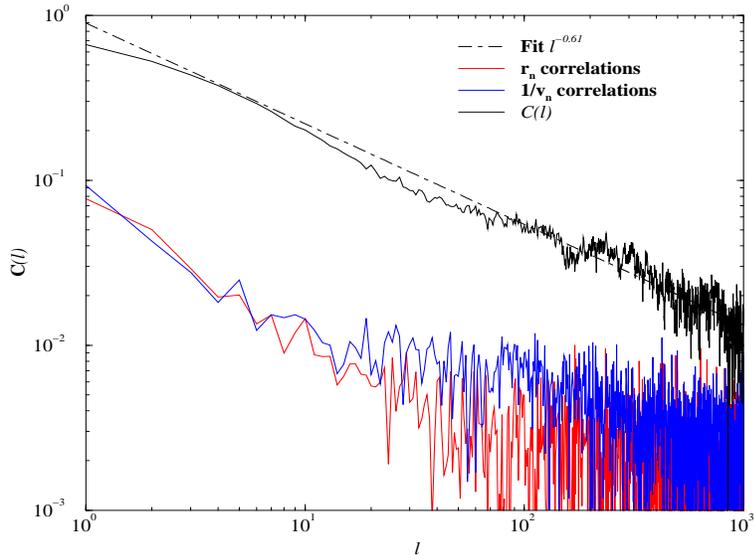


Figure 5: Sign correlations $\mathcal{C}(\ell)$ for BN, showing a long range, power-law decay, and comparison between the smaller and faster decaying correlation of the r_n and the $1/v_n$, showing that the former is dominated by the weak correlations between small order volumes, and not by a compensation between market order flows and limit order flows.

of the dynamics of prices that allows one to reveal subtle effects, beyond the simple autocorrelation of price changes (see also below).

Finally, we show in Fig. 5 the rapid fall of the autocorrelation of the variables r_n , that was argued by Lillo and Farmer to be a strong support to their model [9]. Unfortunately, this effect is not relevant and is due to the fact that the volume at the best price has large fluctuations. For example, in the case of FTE, the distribution of v is found to be well-fit by $P(v) \propto v^{\mu-1} \exp(-v/v_0)$ with $\mu > 1$, so that the most probable values correspond to $v \sim 1$, whereas the mean value is ~ 3000 [11]. Since v_n appears in the denominator of r_n , it is clear that the r_n correlations are dominated by times where the volume at bid/ask is particularly small; these small values show little autocorrelations (see Fig. 5).²

2.3 The bare impact function and price diffusion

We conclude from Fig. 4 that although the variables r_n are indeed close to being uncorrelated, they do not provide an adequate basis to interpret the dynamics of real price. Our transient impact model, on the other hand, allows one to reconcile the absence of autocorrelations in price changes with the observed non

²After discussions, Lillo and Farmer have agreed that their results on LSE stocks are in fact compatible with the above interpretation.

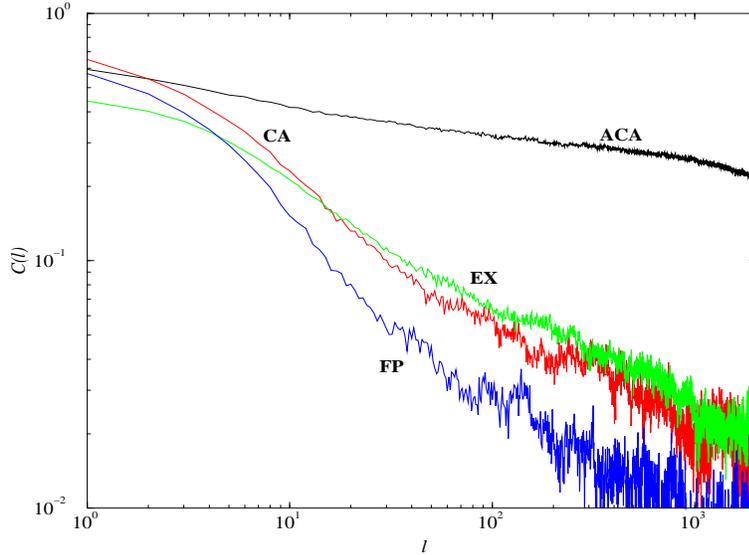


Figure 6: Plot of the sign correlations $\mathcal{C}(\ell)$ for a selection of four stocks, showing the long-ranged nature of these correlations. See also Table 3.

monotonous shape of the average response function, provided the bare impact function $G_0(\ell)$ is chosen adequately. In [6], it was shown that if the correlation of the ε 's decays as $\ell^{-\gamma}$, then $G_0(\ell)$ should also decay, at large times, as a power-law $\ell^{-\beta}$ with $\beta \approx (1 - \gamma)/2$. For $\beta > (1 - \gamma)/2$, the price is subdiffusive (anti-persistent) and the response function $\mathcal{R}(\ell)$ has a maximum before becoming negative at large ℓ . For $\beta < (1 - \gamma)/2$, on the other hand, the price is superdiffusive (persistent) and the response function monotonously increases (see Fig. 10 of [6]). The short time behaviour of $G_0(\ell)$ can in fact be extracted from empirical data by using the following exact relationship:

$$\mathcal{R}(\ell) = \langle \ln V \rangle G_0(\ell) + \sum_{0 < n < \ell} G_0(\ell - n) \mathcal{C}(n) + \sum_{n > 0} [G_0(\ell + n) - G_0(n)] \mathcal{C}(n). \quad (9)$$

where:

$$\mathcal{C}(\ell) = \langle \varepsilon_{n+\ell} \varepsilon_n \ln V_n \rangle, \quad (10)$$

a correlation function that can also be measured directly (see Figs. 5,6).

Eq. (9) gives a set of linear equations relating \mathcal{R} , G_0 and \mathcal{C} that can easily be solved for G_0 . The result is plotted in Fig. 7 for different stocks. One sees that $G_0(\ell)$ is first flat or rises very slightly with ℓ before indeed decaying, for $\ell \gg 1$, like a power law, with β given in Table 2. The fit used to extract the value of β is $G_0^f(\ell) = \Gamma_0 / (\ell_0^2 + \ell^2)^{\beta/2}$ which is similar, but not identical to, the one proposed in [6]. The advantage of the present fit is that it matches quite well the rather flat initial behaviour of $G_0(\ell)$. We also give in Table 2 the value of other quantities such as the exponent γ governing the decay of the ε correlations. A very similar

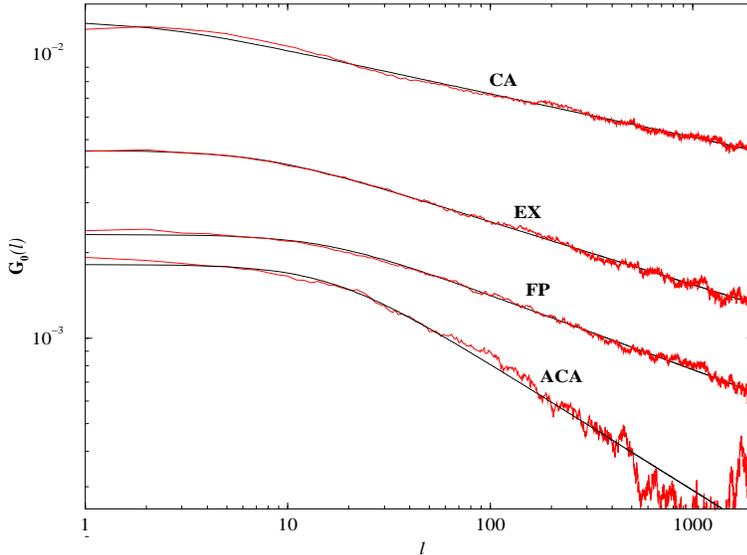


Figure 7: Comparison between the empirically determined $G_0(\ell)$, extracted from \mathcal{R} and \mathcal{C} using Eq.(9), and the fit $G_0^f(\ell) = \Gamma_0/(\ell_0^2 + \ell^2)^{\beta/2}$, used to extract the parameters given in Table 2, for a selection of four stocks: ACA, CA, EX, FP.

shape for G_0 can be observed for all stocks; fluctuations around the critical line $\beta = (1 - \gamma)/2$ (see Fig. 8) are enough to explain the fact that \mathcal{R} sometimes has a maximum, sometimes not.

Correspondingly, the vicinity of the critical line ensures that the price has a diffusive behaviour, as is indeed confirmed by measuring the variance of price changes:

$$\mathcal{D}(\ell) = \langle (p_{n+\ell} - p_n)^2 \rangle \approx D\ell; \quad \forall \ell, \quad (11)$$

as demonstrated in Figs. 9 and 10. The fact that $\mathcal{D}(\ell)$ is strictly linear in ℓ is of course tantamount to saying that price increments are uncorrelated.

2.4 Economic interpretation of the shape of the bare impact

The economic interpretation of the non monotonic behaviour of G_0 is as follows. Suppose that you are a liquidity provider, making profits on the bid-ask spread and losses on large price excursions, and that you see a flow of buy orders coming. In the absence of news and for typical buy volumes,³ the natural strategy is, on short times, to bias the ask price up to be able to sell higher while there are

³The following discussion is intended to describe typical situations. Obviously, if the buy volume is anomalously large, liquidity providers would anticipate some insider information and react differently.

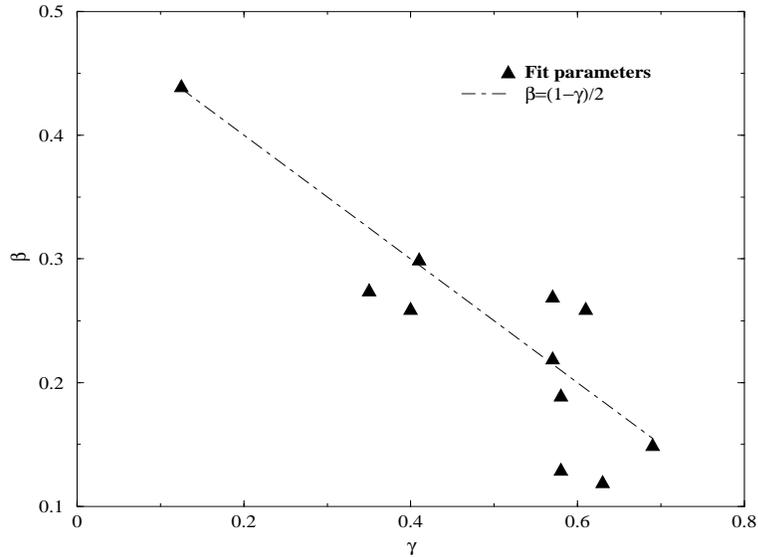


Figure 8: Scatter plot of the exponents β, γ extracted from the fit of G_0 and \mathcal{C} . These exponents are seen to lie in the vicinity of the critical line $\beta = (1 - \gamma)/2$ (dotted line), as expected from the nearly diffusive behaviour of prices (see Fig. 9), and [6].

Stock	$\sqrt{D(1)}$	Γ_0	ℓ_0	β	C_0	γ	Z
ACA	1.69	0.63	16.3	0.44	0.58	0.125	0.35
BN	7.9	1.75	3.1	0.26	0.81	0.61	0.37
CA	3.13	0.71	7.4	0.22	0.83	0.57	0.27
CGE	0.84	0.20	8.9	0.275	0.49	0.35	0.18
EN	2.75	0.66	9.2	0.27	0.83	0.57	0.27
EX	1.79	0.47	15.3	0.26	0.45	0.40	0.20
FP	7.0	1.46	2.2	0.15	0.79	0.69	0.28
FTE	3.9	0.47	20.3	0.30	0.52	0.41	0.23
GLE	4.37	0.73	0.7*	0.13	0.86	0.58	0.28
MC	3.47	0.67	3.1	0.19	0.95	0.58	0.26
VIE	2.8	0.38	0.25*	0.12	0.75	0.63	0.26

Table 2: Summary of the different quantities and fit parameters for 11 stocks of the Paris Bourse during the year 2002. $G_0(\ell)$ is fitted as: $G_0(\ell) = \Gamma_0/(\ell_0^2 + \ell^2)^{\beta/2}$, and $\mathcal{C}(\ell) = C_0/\ell^\gamma$, both in the range $\ell = 2 \rightarrow 2000$. $\sqrt{D(1)}$ and Γ_0 are in cents of Euros. The * means that the fit of G_0 for small ℓ is not very good. The last column measures the *relative* explicative power of the Lillo-Farmer variable, compared to our own: $Z = \mathcal{R}_{LF}(1)/\mathcal{R}(1)$.

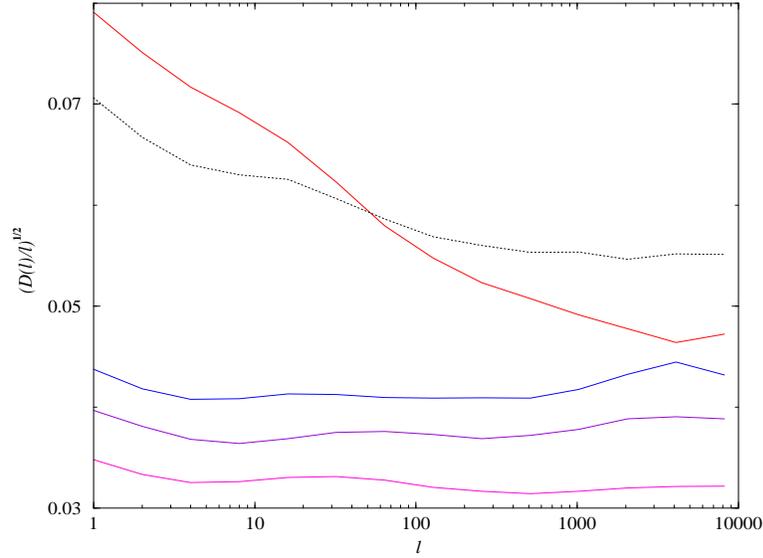


Figure 9: Plot of $\sqrt{\mathcal{D}(\ell)}/\ell$ (in Euros) vs. ℓ for several stocks. Apart from BN and FP (for which the tick size is large), this quantity is roughly constant with ℓ , showing that prices are to a very good approximation diffusive, even on shortest times scales. From top to bottom: BN, FP, GLE, FTE, MC.

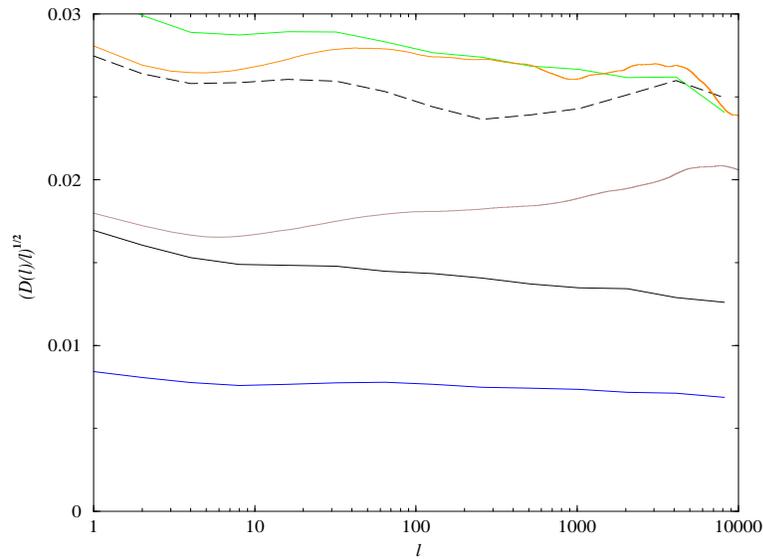


Figure 10: Plot of $\sqrt{\mathcal{D}(\ell)}/\ell$ (in Euros) vs. ℓ for all other (smaller tick) stocks. From top to bottom: CA, VIE, EN, EX, ACA, CGE.

clients eager to buy. However, you now have a net short position on the stock that you want to eventually shift back to zero. So you would like to buy back, in the near future, at the cheapest possible price. In order to prevent the price from going up, you can/should do two things: a) create a barrier to further price rises by placing a large number of sell orders at the ask, off which the price will bounce back down b) place bid orders as low as possible. Both effects act to create a liquidity molasses that mean revert the price towards its initial value. *Both* these effects can actually be observed directly on the data.

- a) One observes a strong correlation between a buy (resp. sell) market order moving the price up and the subsequent appearance of limit orders at the ask (resp. bid) [17, 9]. If a ‘wall’ of limit orders appears at the ask while the bid remains poorly populated, the probability that the price moves down upon the arrival of further market orders becomes larger than the probability to move up. One can visualize this effect more clearly by separating the total price change into two components: price variations due to market orders, $\Delta_M p_n$, corresponding to the change of mid-point between the quote immediately prior and the quote immediately posterior to the n -th trade, and price variations due to limit orders, $\Delta_L p_n$ corresponding to changes of mid-points in-between trade n and trade $n + 1$. By definition,

$$p_{n+\ell} - p_n = \sum_{k=n}^{n+\ell-1} [\Delta_M p_k + \Delta_L p_k] \equiv (p_{n+\ell} - p_n)_M + (p_{n+\ell} - p_n)_L. \quad (12)$$

One can then measure the response function restricted to price changes due to market orders:

$$\mathcal{R}_M(\ell) = \langle (p_{n+\ell} - p_n)_M \cdot \varepsilon_n \rangle, \quad (13)$$

and compare it (see Fig. 11) to $\mathcal{R}(\ell)$. We observe for all stocks that $\mathcal{R}_M(\ell)$ and $\mathcal{R}(\ell)$ have the same overall shape. For FTE, for example, $\mathcal{R}_M(\ell)$ also bends down and becomes negative for large ℓ . But since by definition $\Delta_M p_k = \varepsilon_k \mathcal{G}_k$ with $\mathcal{G}_k \geq 0$ (a buy market order can only move the price up or leave it unchanged), the fact that $\mathcal{R}_M(\ell)$ decreases implies that \mathcal{G}_k is *anticorrelated* with $\varepsilon_n \varepsilon_k$. In other words, sell orders following buy orders impact the price more than buy orders following buy orders, as expected if the order book fills in more on the ask side than on the bid side after a buy market order (and, of course, similarly for the sell side).

- b) there is an anticorrelation between buy orders and the subsequent motion of the bid-ask *in-between* trades. This is seen both from the fact that $\mathcal{R}_M(\ell) > \mathcal{R}(\ell)$ for ℓ not too large (see Fig. 11), implying that the response function restricted to limit orders is negative. Furthermore, one can study the correlation between a market order induced price change $\Delta_M p_n$ and a later limit order price change $\Delta_L p_{n+\ell}$, which is found to be negative (as

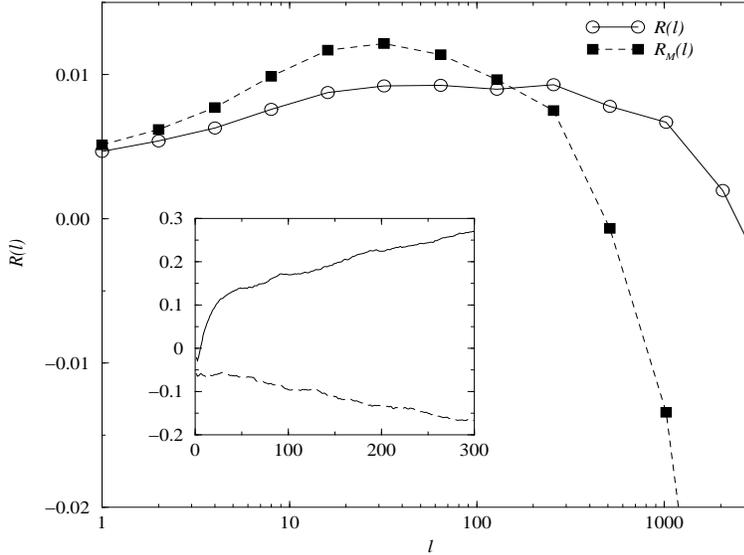


Figure 11: Main figure: Comparison between the full response \mathcal{R} (circles) and the response restricted to market order induced price changes \mathcal{R}_M (squares), for FTE in 2002. Inset: Integrated correlation functions, corresponding to $\langle \Delta_M p_n \cdot \Delta_M p_{n+l} \rangle$ (full line), and $\langle \Delta_M p_n \cdot \Delta_L p_{n+l} \rangle$ (dotted line). The former is clearly positive, and is compensated by the negative correlation between market orders induced shifts and subsequent changes in the mid-quotes.

also reported in [6, 9]). This compensates the positive correlations between $\Delta_M p_n$ and $\Delta_M p_{n+l}$ (and between $\Delta_L p_n$ and $\Delta_L p_{n+l}$), that would otherwise lead to a superdiffusion in the price.

In order to make our point even more clearly, it is useful to emphasize the antagonist forces present in financial markets:

- The ideal world for liquidity providers is a stable, fixed average price that allows them to earn the bid-ask spread at every round-turn. Volatility is the enemy⁴, liquidity molasses is the solution: a vanishing long term impact (i.e. $G_0(\infty) = 0$) is a way to limit the volatility of the market and to increase the liquidity provider gains. Reducing the volatility of financial markets is in fact the traditional role given to market makers in non electronic markets. Note that we do not assume any kind of collusion between liquidity providers: they all, individually, follow a perfectly reasonable strategy.

⁴Insider information is also the liquidity provider enemy, but this situation is rather rare on the scale of the thousands of trades happening every day on each single liquid stock. However, creating a liquidity wall is indeed risky for the liquidity provider in the case where some true information motivates the market orders. In that case, the insider can use his information without impacting the price.

- Conversely, *permanent* impact is what the liquidity taker should hope for: if the price rises because of his very trade but stays high until he sells back, his impact is not really a cost. On the other hand, if the price deflates back after having bought it, it means that he paid too much for it.⁵ The correlations created by splitting his bid in small quantities also help keeping the price up.

These are the basic ingredients ruling the competition between liquidity providers and liquidity takers. The subtle balance between the positive correlation in the trades (measured by γ) and the liquidity molasses induced by liquidity providers (measured by β) is a *self-organized dynamical equilibrium*. Its stability comes from two counter-balancing effects: if the liquidity providers are too slow to revert the price ($\beta < (1 - \gamma)/2$), then the price is superdiffusive and liquidity providers lose money on average [21]; therefore they increase β . If the mean reversion is too strong ($\beta > (1 - \gamma)/2$), the resulting long term anticorrelations is an incentive for buyers to wait for prices to come back down to continue buying. Liquidity takers thereby spread their trading over longer time scales, which corresponds to smaller values of γ .

A dynamical equilibrium where $\beta \approx (1 - \gamma)/2$ therefore establishes itself spontaneously, with clear economic forces driving the system back towards this equilibrium. Interestingly, *fluctuations* around this critical line leads to fluctuations of the local volatility, since persistent patches correspond to high local volatility and antipersistent patches to low local volatility (see also [22] for a similar mechanism). Extreme crash situations are well-known to be liquidity crisis, where the liquidity molasses effect disappears temporarily, destabilising the market (on that point, see the detailed recent study of [12, 18]).

Finally, the mean-reverting nature of the response function is of crucial importance to understand the influence of volume and execution time on the actual impact of trading on prices (on this point, see [19, 20]).

3 Summary and Conclusion

The aim of this paper was to challenge Lillo and Farmer's suggestion that the strong memory in the signs of trades is compensated by liquidity fluctuations, with an otherwise *permanent* market impact, and confirm the more subtle scenario proposed in our previous paper [6], in which the impact is *transient*, with a power-law decay in time. The exponent is precisely tuned to a critical value, ensuring simultaneously that prices are diffusive on long time scales and that the response function is nearly constant. Therefore, the seemingly trivial random walk behaviour of price changes in fact results from a finely-tuned competition

⁵The salesman knows nothing about what he is selling, save that he is charging a great deal too much for it. (Oscar Wilde)

between two opposite effects, one leading to super-diffusion – the autocorrelation of market order flow; the other leading to sub-diffusion – the decay of the bare impact function, reflecting the mean-reverting nature of the limit order flow. We have shown that mean reversion comes both from an excess flow of limit order opposite to the market order flow, and to a systematic anti-correlation of the bid-ask motion between trades. Note that in the above picture, the random walk nature of prices and their volatility are induced by the trading mechanisms alone, with no reference to real news. These of course should also play a role, but probably not as important as pure speculation and trading that lead to excess volatility (see the discussion and references in [6]).

The above fine tuning is however, obviously, not always perfect, and is expected to be only approximately true on average. Breakdown of the balance between the two effects can lead either to large volatility periods and crashes when the liquidity molasses disappears, or to low volatility periods when mean-reverting effects are strong. The small imbalance between the two effects therefore leads to different shapes of $\mathcal{R}(\ell)$ (monotone increasing or turning round and changing sign). As emphasized in [6], our finding that the absence of arbitrage opportunities results from a critical balance between antagonist effects might justify several claims made in the (econo-)physics literature that the anomalies in price statistics (fat tails in returns described by power laws [23, 24], long range self similar volatility correlations [3, 5], and the long ranged correlations in signs [6, 9]) are due to the presence of a critical point in the vicinity of which the market operates (see e.g. [25], and in the context of financial markets [26, 27, 28]). From a more practical point of view, we hope that the present detailed picture of market microstructure could help understanding the mechanisms leading to excess volatility, and suggest ways to control more efficiently the stability of financial markets.

Acknowledgments

We want to thank Matthieu Wyart and Yuval Gefen for many inspiring discussions and ideas about this work. We also thank Doyne Farmer and Fabrizio Lillo for many comments and e-mail exchanges that allowed to clarify a lot the present paper.

References

- [1] R. J. Shiller, *Do Stock Prices move too much to be justified by subsequent changes in dividends ?*, American Economic Review, **71**, 421 (1981). R. J. Shiller, *Irrational Exuberance*, Princeton University Press (2000).

- [2] B. LeBaron, *Stochastic volatility as a simple generator of apparent financial power laws and long memory*, Quantitative Finance, **1** 621 (2001).
- [3] R. Cont, *Empirical properties of asset returns: stylized facts and statistical issues*, Quantitative Finance, **1**, 223, (2001).
- [4] J.-P. Bouchaud and M. Potters, *Théorie des Risques Financiers*, Aléa-Saclay, 1997; *Theory of Financial Risks and Derivative Pricing*, Cambridge University Press, 2000 and 2004.
- [5] J.-F. Muzy, J. Delour, E. Bacry, *Modelling fluctuations of financial time series: from cascade process to stochastic volatility model*, Eur. Phys. J. B **17**, 537-548 (2000); for a recent review, see: J.P. Bouchaud, J.F. Muzy, *Financial time series: from Bachelier's random walk to multifractal 'cascades'*, in 'Kolmogorov's Legacy', Editions Belin/Springer, Paris/Berlin 2003.
- [6] J.P. Bouchaud, Y. Gefen, M. Potters, M. Wyart, *Fluctuations and Response in Financial Markets: The subtle nature of 'random' price changes*, Quantitative Finance **4**, 176 (2004).
- [7] J. Hasbrouck, *Measuring the information content of stock trades*, Journal of Finance, **XLVI**, 179 (1991).
- [8] C. Hopman, *Are supply and demand driving stock prices?*, MIT working paper, Dec. 2002.
- [9] F. Lillo, J. D. Farmer, *The long memory of efficient markets*, e-print cond-mat/0311053.
- [10] M. G. Daniels, J. D. Farmer, G. Iori, E. Smith, *Quantitative model of price diffusion and market friction based on trading as a mechanistic random process*, Phys. Rev. Lett. **90**, 108102 (2003). E. Smith, J. D. Farmer, L. Gillemot, S. Krishnamurthy, *Statistical theory of the continuous double auction*, Quantitative Finance **3**, 481 (2003).
- [11] J.P. Bouchaud, M. Mézard, M. Potters, *Statistical properties of stock order books: empirical results and models*, Quantitative Finance **2**, 251 (2002).
- [12] J. Dooyne Farmer, Laszlo Gillemot, Fabrizio Lillo, Szabolcs Mike, Anindya Sen, *What really causes large price changes?*, e-print cond-mat/0312703.
- [13] V. Plerou, P. Gopikrishnan, X. Gabaix, H. E. Stanley, *Quantifying Stock Price Response to Demand Fluctuations*, Phys. Rev. E **66**, 027104 (2002).
- [14] F. Lillo, R. Mantegna, J. D. Farmer, *Single Curve Collapse of the Price Impact Function for the New York Stock Exchange*, e-print cond-mat/0207428; *Master curve for price-impact function*, Nature, **421**, 129 (2003).

- [15] F. Lillo, J. D. Farmer, *On the origin of power-law tails in price fluctuations*, e-print cond-mat/0309416
- [16] M. Potters, J.P. Bouchaud, *More statistical properties of order books and price impact*, Physica A, **324**, 133 (2003).
- [17] P. Weber, B. Rosenow, *Order book approach to price impact*, e-print cond-mat/0311457
- [18] P. Weber, B. Rosenow, *Large stock price changes: volume or liquidity?*, e-print cond-mat/0401132
- [19] X. Gabaix, P. Gopikrishnan, V. Plerou, H. E. Stanley, *A theory of power-law distributions in financial markets fluctuations*, Nature, **423**, 267 (2003).
- [20] J. Doyne Farmer, F. Lillo, *On the origin of power-law tails in price fluctuations*, Quantitative Finance, **4** C7 (2004), V. Plerou, P. Gopikrishnan, X. Gabaix, H. E. Stanley, *On the origin of power-law fluctuations in stock prices*, Quantitative Finance, **4** C11 (2004).
- [21] M. Wyart, J.P. Bouchaud, in preparation.
- [22] M. Wyart, J.P. Bouchaud, *Self-referential behaviour, overreaction and conventions in financial markets*, e-print cond-mat/0303584, submitted to J. Economic Behaviour & Organisation.
- [23] T. Lux, *The stable Paretian hypothesis and the frequency of large returns: an examination of major German stocks*, Applied Financial Economics, **6**, 463, (1996).
- [24] V. Plerou, P. Gopikrishnan, L.A. Amaral, M. Meyer, H.E. Stanley, *Scaling of the distribution of price fluctuations of individual companies*, Phys. Rev. **E60** 6519 (1999); P. Gopikrishnan, V. Plerou, L. A. Amaral, M. Meyer, H. E. Stanley, *Scaling of the distribution of fluctuations of financial market indices*, Phys. Rev. **E 60** 5305 (1999).
- [25] P. Bak, *How Nature Works: The Science of Self-Organized Criticality*, Copernicus, Springer, New York, 1996.
- [26] D. Challet, A. Chessa, M. Marsili, Y.C. Zhang, *From Minority Games to real markets*, Quantitative Finance, **1**, 168 (2001) and refs. therein; D. Challet, M. Marsili, Y.C. Zhang, *The Minority Game*, book in preparation.
- [27] T. Lux, M. Marchesi, *Volatility Clustering in Financial Markets: A Microsimulation of Interacting Agents*, Int. J. Theo. Appl. Fin. **3**, 675 (2000).

- [28] I. Giardina, J. P. Bouchaud, *Bubbles, crashes and intermittency in agent based market models*, Eur. Phys. Jour. B **31**, 421 (2003); see also the discussion in: J.P. Bouchaud, *Power-laws in economics and finance: some ideas from physics*, Quantitative Finance, **1**, 105 (2001).