Smile dynamics – a theory of the implied leverage effect:

ERRATUM

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We discovered a very unfortunate mistake in our paper “Smile dynamics – a theory of
the implied leverage effect” [1], where the predictions for the change of implied volatility for
a fixed strike and for a fixed moneyness got mixed up. When the return of the underlying
is \( r \), the theory predicts that the at-the-money (ATM) implied volatility \( \Sigma_t(\mathcal{M} = 0, T) \) for
fixed moneyness should evolve as:

\[
\frac{\delta \Sigma(0, T)}{\Sigma(0, T)} \approx \frac{r}{2 \Sigma(0, T) T} \int_0^T du \ g_L(u) = \gamma(T)r, \tag{1}
\]

where \( g_L \) is the leverage correlation function of returns \( g_L(t) = \langle r_i t^2 \rangle_c / \sigma^3 \). This expression
should be compared to the original expression in [1]:

\[
\frac{\delta \Sigma(0, T)}{\Sigma(0, T)} \approx \frac{r}{2 \Sigma(0, T) T^2} \int_0^T du \ u \ g_L(u), \tag{2}
\]

which instead holds for the implied volatility of a fixed strike option with a moneyness close
to zero.

The correct comparison with empirical data on the OEX index, large cap, mid cap and
small cap stocks, is given in Figs. 1-4, where we show the implied leverage coefficient \( \gamma(T) \)
as a function of maturity. The implied data is obtained by regressing the relative daily
change of ATM implied vols on the corresponding stock or index return, for each maturity.
The result is then averaged over all stocks within a given tranche of market capitalisation.
The three curves correspond to (a) the correct theoretical prediction computed using the
historically determined leverage correlation \( g_L(t) \); (b) the original theoretical prediction; (c)
the “sticky strike” procedure. We also show the \( \gamma = 0 \) line corresponding to “sticky delta”.

We see that for the OEX index, the implied volatility overreacts to changes of prices
compared to the prediction calibrated on the historical leverage effect, except for the shortest
maturities where the prediction is right on the empirical value. For single stocks, small
maturity options tend to underreact, whereas longer maturities tend to overreact. As men-
tioned in [1], the empirical curves for the OEX and for large caps appear to be well fitted
by a sticky-strike prediction, but with amplitude of the leverage correlation substantially
larger than its historical value. This would be compatible with the fact that market makers
use a simple sticky strike procedure, but with a smile that is significantly more skewed than
justified by historical data, or else that the a local volatility model is used, since this leads
to a factor 2 amplification of the sticky-strike prediction [2].

The other results of the paper, in particular the empirical work, is unaffected by the
above blunder. We thank V. Vargas and L. De Leo for discussions.
FIG. 1: Comparison between the implied leverage coefficient $\gamma$ and the various theoretical predictions, sticky strike, strike delta, historical (wrong) and historical (corrected), for large cap US stocks in the period 2004-2008.

FIG. 2: Same as Fig. 1, but for mid-cap US stocks.

FIG. 3: Same as Fig. 1, but for small-cap US stocks.

FIG. 4: Same as Fig. 1, but for the OEX index. Note that the shortest, most liquid, maturity is perfectly explained by the theory, whereas longer maturities overreact.