

# The leverage effect in financial markets: retarded volatility and market panic

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## Abstract

We investigate quantitatively the so-called leverage effect, which corresponds to a negative correlation between past returns and future volatility. For individual stocks, this correlation is moderate and decays exponentially over 50 days, while for stock indices, it is much stronger but decays faster. For individual stocks, the magnitude of this correlation has a universal value that can be rationalized in terms of a new 'retarded' model which interpolates between a purely additive and a purely multiplicative stochastic process. For stock indices a specific market panic phenomenon seems to be necessary to account for the observed amplitude of the effect.

## 1 Introduction

Several 'stylized fact' of financial markets, such as 'fat tails' in the distribution of returns or long ranged volatility correlations, have recently become the focus of detailed empirical study [1, 2, 3, 4, 5, 6, 7, 8, 9]. Simple agent based models have been proposed, with some degree of success, to explain these features [10, 11, 12, 13, 14, 15, 16]. Another well-known stylized fact is the so-called 'leverage' effect, first discussed by Black [18, 19], who observed that the volatility of stocks tends to increase when the price drops. This effect is particularly important for option markets: not only does it imply that at-the-money volatilities tend to increase after price drops, but also that a significant skew in the volatility smile should appear [21, 7, 20, 22], as is indeed observed on markets where the leverage effect

is strong. This skew reflects the fact that a negative volatility-return correlation induces a negative skew in the distribution of price returns themselves [21, 20].

Although widely discussed in the economic and econometric literature [3, 17], the leverage effect (or volatility-return correlation) has been less systematically investigated than the volatility clustering effect (volatility-volatility correlation). For example, one would like to know if the volatility-return correlation shows a long term dependence similar to that observed on the volatility-volatility correlation. Although various single correlation coefficients quantifying the leverage effect have been measured and discussed within GARCH models [3, 17], the full temporal structure of this correlation has not been quantitatively investigated. The economic interpretation of this leverage effect is furthermore still controversial; a recent survey of the different models can be found in [17]. Even the causality of the effect is debated [17]: is the volatility increase induced by the price drop or conversely do prices tend to drop after a volatility increase?

In the present paper, we report some empirical study of this leverage effect both for individual stocks and for stock indices. We find unambiguously that correlations are between future volatilities and past price changes. For both stocks and stock indices, the volatility-return correlation is short ranged, with however a different decay time for stock indices (around 10 days) and for individual stocks (around 50 days). The amplitude of the correlation is also different, and is much stronger for indices than for stocks. We then argue that the leverage effect for stocks can be interpreted within a simple retarded model, where the absolute amplitude of price changes does not follow the price level instantaneously (as is assumed in most models of price changes, such as the geometric Brownian motion). Rather, absolute price changes are related to an average level of the past price. This reflects the lag with which market operators change their behavior (order volumes, bid-ask spreads, transaction costs, etc.) when the price evolves: the proportionality between absolute price changes and the level of the price itself is only ensured on the long run. We then show that this model does not represent adequately the leverage effect for stock indices, which seems to reflect a ‘panic’-like effect, whereby large price drops of the market as a whole triggers a significant increase of activity.

## 2 Empirical results

We will call  $S_i(t)$  the price of stock  $i$  at time  $t$ , and  $\delta S_i(t)$  the (absolute) daily price change:  $\delta S_i(t) = S_i(t+1) - S_i(t)$ . The relative price change will be denoted as  $\delta x_i(t) = \delta S_i(t)/S_i(t)$ . The leverage correlation function which naturally appears in the calculation of the skewness of the distribution of price changes over a horizon  $T$  is [20]:

$$\mathcal{L}_i(\tau) = \frac{1}{Z} \left\langle [\delta x_i(t+\tau)]^2 \delta x_i(t) \right\rangle, \quad (1)$$

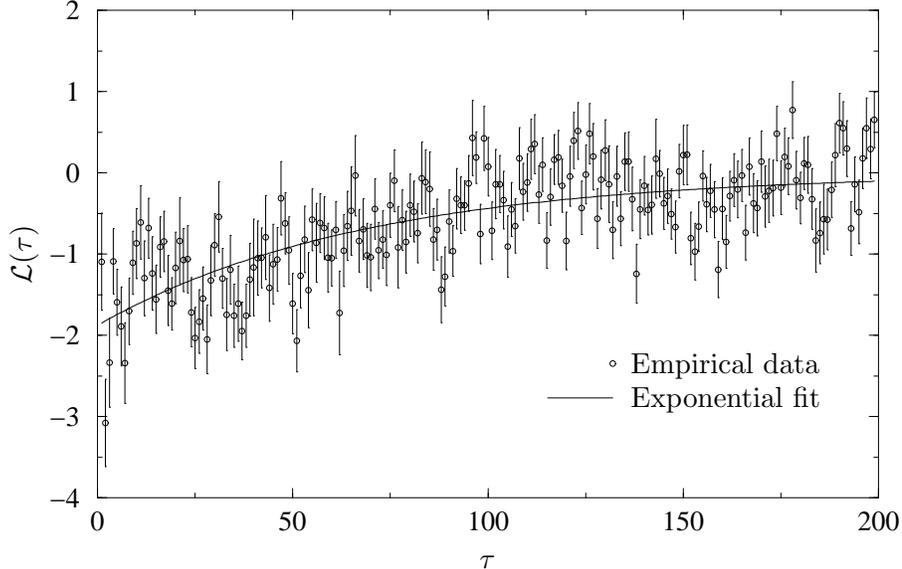


Figure 1: Return-Volatility correlation for individual stocks. Data points are the empirical correlation averaged over 437 US stocks, the error bars are two sigma errors bars estimated from the inter-stock variability. The full line shows an exponential fit (Eq. (3)) with  $A_S = 1.9$  and  $T_S = 69$  days. Note that  $\mathcal{L}(\tau = 0)$  is not far from  $-2$ , as our retarded model predicts.

which measures the correlation between price change at time  $t$  and a measure of the square volatility at time  $t + \tau$ . The coefficient  $Z$  is a normalization that we have chosen to be:

$$Z = \langle \delta x_i(t)^2 \rangle^2 \quad (2)$$

for reasons that will become clear below. (Note that since  $\delta x$  is dimensionless,  $\mathcal{L}_i(\tau)$  is also dimensionless, despite this apparent lack of homogeneity between the numerator and denominator.)

In the following, we will consider  $\tau \geq 1$ . Negative values of  $\tau$  lead to very small values of the correlation function, indistinguishable from noise. In other words, the correlation exists between future volatilities and past price changes; conversely, volatility changes do not convey any useful information on future price changes.

We have analyzed a set of 437 US stocks, constituent of the S&P 500 index and a set of 7 major international stock indices. Our dataset consisted of daily data ranging from Jan. 1990 to May 2000 for stocks and from Jan. 1990 to Oct. 2000 for indices. We have computed  $\mathcal{L}_i$  both for individual stocks and stock indices. The raw results were rather noisy. We have therefore assumed that individual stocks behave similarly and averaged  $\mathcal{L}_i$  over the 437 different stocks in our dataset to give  $\overline{\mathcal{L}}_S$ , and over 7 different indices (from the US, Europe and

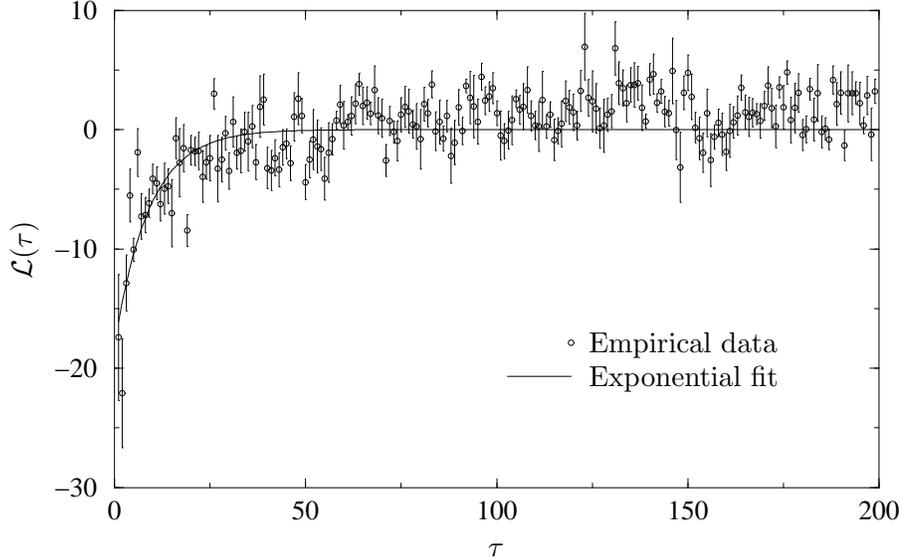


Figure 2: Return-Volatility correlation for stock indices. Data points are the empirical correlation averaged over 7 major stock indices, the error bars are two sigma errors bars estimated from the inter-index variability. The full line shows an exponential fit (Eq. (3)) with  $A_I = 18$  and  $T_I = 9.3$  days.

Asia), to give  $\overline{\mathcal{L}}_I$ . The results are given in Fig. 1 and Fig. 2 respectively. As can be seen from these figures, both  $\overline{\mathcal{L}}_S$  and  $\overline{\mathcal{L}}_I$  are clearly negative: price drops increase the volatility – this is the so-called leverage effect. These correlation functions can rather well be fitted by *single* exponentials:

$$\overline{\mathcal{L}}_{S,I}(\tau) = -A_{S,I} \exp\left(-\frac{\tau}{T_{S,I}}\right). \quad (3)$$

For US stocks, we find  $A_S = 1.9$  and  $T_S \simeq 69$  days, whereas for indices the amplitude  $A_I$  is significantly larger,  $A_I = 18$  and the decay time shorter:  $T_I \simeq 9$  days. This exponential decay should be contrasted with the very slow, power-law like decay of the volatility correlation function, which cannot be characterized by a unique decay time [2, 4, 5, 6, 7, 8, 9]. Therefore, a new time scale seems to be present in financial markets, intermediate between the very high frequency time scale seen on the correlation function of returns (several minutes) and the very low frequency time scales appearing in the volatility correlation function.

### 3 A retarded model

Traditional models of asset fluctuations postulate that price changes are proportional to prices themselves. The price increment is therefore written as:

$$\delta S_i(t) = S_i(t) \sigma_i \epsilon(t), \quad (4)$$

where  $\sigma_i$  is the volatility and  $\epsilon$  a random variable with zero mean and unit variance, independent of all past history [23]. Eq. (4) means that price increments are at any time proportional to the current value of the price. Although it is true that on the long run, price increments tend to be proportional to prices themselves, this is not reasonable on short time scales. Locally, prices evolve in discrete steps (ticks), following buy or sell orders that can only be expressed an integer number of contracts. The mechanisms leading to price changes are therefore not expected to vary continuously as prices evolve, but rather to adapt only progressively if prices are seen to rise (or decrease) significantly over a certain time window. The model we propose to describe this lagged response to price changes is to replace  $S_i$  in Eq. (4) by a moving average  $S_i^R$  over a certain time window. More precisely, we write:

$$\delta S_i(t) = S_i^R(t) \sigma_i \epsilon, \quad S_i^R(t) = \sum_{\tau=0}^{\infty} \mathcal{K}(\tau) S_i(t - \tau), \quad (5)$$

where  $\mathcal{K}(\tau)$  is a certain averaging kernel, normalized to one:

$$\sum_{\tau=0}^{\infty} \mathcal{K}(\tau) \equiv 1. \quad (6)$$

For example, an exponential moving average corresponds to  $\mathcal{K}(\tau) = (1 - \alpha)\alpha^\tau$ , ( $\alpha < 1$ ). It will be more congenial to rewrite  $S_i^R$  as:

$$S_i^R(t) = \sum_{\tau=0}^{\infty} \mathcal{K}(\tau) \left[ S_i(t) - \sum_{\tau'=1}^{\tau} \delta S_i(t - \tau') \right] = S_i(t) - \sum_{\tau'=1}^{\infty} \bar{\mathcal{K}}(\tau') \delta S_i(t - \tau'), \quad (7)$$

where  $\bar{\mathcal{K}}(\tau)$  is the (discrete) integral of  $\mathcal{K}(\tau)$ :

$$\bar{\mathcal{K}}(\tau) = \sum_{\tau'=\tau}^{\infty} \mathcal{K}(\tau'). \quad (8)$$

Note that from the normalization of  $\mathcal{K}(\tau)$ , one has  $\bar{\mathcal{K}}(0) = 1$ , independently of the specific shape of  $\mathcal{K}(\tau)$ . This will turn out to be crucial in the following discussion.

For the exponential model, one finds  $\bar{\mathcal{K}}(\tau) = \alpha^\tau$ . The limit  $\alpha \rightarrow 1$  corresponds to the case where  $S_i^R(t)$  is a constant, and therefore Eq. (5) corresponds to an *additive* model. The other limit  $\alpha \rightarrow 0$  (infinitely small averaging time window) leads to  $S_i^R(t) \equiv S_i(t)$  and corresponds to a purely *multiplicative* model. An value of  $\alpha$  close to one,  $\alpha = 1 - \epsilon$  corresponds to an additive model on short time scales ( $\ll T = \epsilon^{-1}$ ) and to a multiplicative model for long time scales ( $\gg T$ ) [7]. A formulation of this model in terms of product of random matrices is given in the Appendix.

In the following, we will assume that the relative difference between  $S_i$  and  $S_i^R$  is small. This is the case when:

$$\eta = \sigma \sqrt{\sum_{\tau'=1}^{\infty} \bar{\mathcal{K}}^2(\tau')} \ll 1 \quad (9)$$

For the exponential model, this is equivalent to  $\sigma\sqrt{T/2} \ll 1$ . In other words,  $\eta \ll 1$  means that the averaging takes place on a time scale such that the relative changes of price are not too large. Typically,  $T = 50$  days,  $\sigma = 2\%$  per square-root day, so that  $\eta \sim 0.1$ .

Let us calculate the correlation function  $\mathcal{L}_i(\tau)$  to first order in  $\eta \ll 1$ . One finds, using  $\delta x_i = \delta S_i/S_i$  and Eq. (5):

$$\langle [\delta x_i(t + \tau)]^2 \delta x_i(t) \rangle = \sigma_i^2 \left\langle \left( 1 - 2 \sum_{\tau'=1}^{\infty} \bar{\mathcal{K}}(\tau') \frac{\delta S_i(t + \tau - \tau')}{S_i(t + \tau)} \right) \frac{\delta S_i(t)}{S_i(t)} \right\rangle. \quad (10)$$

To first order in  $\eta$ , one can replace in the above expression  $\delta S_i(t + \tau - \tau')/S_i(t + \tau)$  by  $\sigma_i \epsilon(t + \tau - \tau')$  and  $\delta S_i(t)/S_i(t)$  by  $\sigma_i \epsilon(t)$ . Since the  $\epsilon$  are assumed to be independent, of zero mean and of unit variance, one has

$$\langle \epsilon(t + \tau - \tau') \epsilon(t) \rangle = \delta_{\tau, \tau'}. \quad (11)$$

Therefore:

$$\langle [\delta x_i(t + \tau)]^2 \delta x_i(t) \rangle = -2\sigma_i^4 \bar{\mathcal{K}}(\tau) \quad (12)$$

With the chosen normalization for  $\mathcal{L}_i(\tau)$  (see Eq. (1)), we finally find:

$$\mathcal{L}_i(\tau) = -2\bar{\mathcal{K}}(\tau) \quad (13)$$

A very important prediction of this model is therefore that  $\mathcal{L}_i(\tau \rightarrow 0) = -2$ . As shown in Fig. 1, this is indeed rather well obeyed for individual stocks, with  $\bar{\mathcal{K}}(\tau)$  given by a simple exponential. We have confirmed this finding by analyzing a set of 500 European stocks and 300 Japanese stocks, again in the period 1990-2000. The results are given in Figure 3 for the European stocks. We again find an exponential behavior with a time scale on the order of 40 days and, more importantly, and initial values of  $\mathcal{L}_i$  close to the retarded model value  $-2$  [24]. A similar graph was obtained for Japanese stocks as well, which is interesting since this market did behave very differently both from the U.S. and European markets during the investigated time period. For the Japanese market, the prediction  $\mathcal{L}(\tau \rightarrow 0) = -2$  not as good: an exponential fit of  $\mathcal{L}(\tau)$  gives and  $A_S = 1.5$  and  $T_S = 47$  days.

We therefore conclude that the leverage effect for stocks might not have a very deep economical significance, but can be assigned to a simple ‘retarded’ effect, where the change of prices are calibrated not on the instantaneous value of the price but on an exponential moving average of the price. On the other hand, as Fig. 2 reveals, the correlation function for indices has a much larger value for  $\tau = 0$  and the above interpretation cannot hold. We will discuss this in more details in the next section.

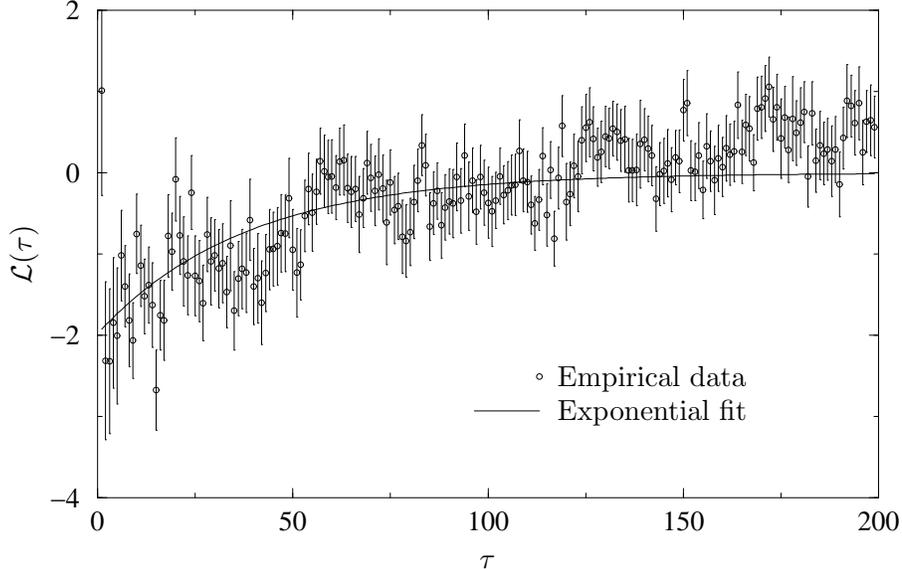


Figure 3: Return-Volatility correlation for European stocks. Data points are the empirical correlation averaged over 500 European stocks. The full line shows an exponential fit (Eq. (3)) with  $A_S = 1.96$  and  $T_S = 38$  days for European stocks. Note again that  $\mathcal{L}$  for small  $\tau$  is very close to the retarded model value  $-2$ .

## 4 A one factor leverage model

Figure 2 shows that (i) the ‘leverage effect’ for indices is much stronger than that appearing for individual stocks, but (ii) tends to decay to zero much faster with the lag  $\tau$ . This is at first sight puzzling, since the stock index is, by definition, an average over stocks. So one can wonder why the strong leverage effect for the index does not show up in stocks and conversely why the long time scale seen in stocks disappears in the index. We want to show in this section that these features can be rationalized by a simple one factor model, where the ‘market factor’ is responsible for the strong leverage effect. Let us therefore write the stock price increments as:

$$\delta S_i(t) = S_i^R(t) [\beta_i \phi(t) + \epsilon_i(t)], \quad (14)$$

where  $\phi(t)$  is the return factor common to all the stocks,  $\beta_i$  are some time independent coefficients, and  $\epsilon_i$  are the so-called idiosyncrasies, uncorrelated from stock to stock and from the common factor  $\phi$ . The market index  $I(t)$  is defined as a certain weighted average of the stocks:

$$I(t) = \sum_{i=1}^N w_i S_i(t), \quad (15)$$

where  $w_i$  are certain weights, of order  $1/N$ . From linearity, one finds the same relation between the ‘retarded’ quantities:

$$I^R(t) = \sum_{i=1}^N w_i S_i^R(t). \quad (16)$$

Neglecting terms of order  $1/\sqrt{N}$ , one finds, after summing Eq. (14) over all  $i$ ’s:

$$\delta I(t) = \bar{\beta} I^R(t) \phi(t), \quad (17)$$

where  $\bar{\beta}$  is the weighted sum of the  $\beta$ ’s:

$$\bar{\beta} = \frac{\sum_{i=1}^N w_i \beta_i S_i^R}{\sum_{i=1}^N w_i S_i^R}, \quad (18)$$

that we will take as a constant. One can always define  $\phi(t)$  such that this constant is unity, which is the choice we make in the following. Now, we postulate that there exists an index specific leverage effect, resulting from an increase of activity when the market as a whole goes down, reflecting a panic like effect. Therefore we write:

$$\left\langle \phi^2(t + \tau) \frac{\delta I(t)}{I(t)} \right\rangle \simeq \left\langle \phi^2(t + \tau) \phi(t) \right\rangle = -\Gamma(\tau) \quad (19)$$

In the following, we will indeed assume that  $\Gamma(\tau)$  is small, as the data suggests, and we neglect all mixed panic-retarded effects. To linear order in the correlations, one then finds the following index leverage effect:

$$\left\langle \left[ \frac{\delta I(t + \tau)}{I(t + \tau)} \right]^2 \frac{\delta I(t)}{I(t)} \right\rangle \simeq -\Gamma(\tau) - 2\bar{\mathcal{K}}(\tau) \left\langle \phi^2(t) \right\rangle^2. \quad (20)$$

Hence, the correctly normalized volatility-return correlation function reads:

$$\mathcal{L}_I(\tau) = -2\bar{\mathcal{K}}(\tau) - \gamma(\tau) \quad (21)$$

with:

$$\Gamma(\tau) = \gamma(\tau) \sigma_I^4 \quad (22)$$

where  $\sigma_I^2 \equiv \sqrt{\langle \phi^2(t) \rangle}$  is the market volatility. Therefore, one explicitly sees that the slowly decaying part  $\bar{\mathcal{K}}(\tau)$  should also appear in  $\mathcal{L}_I(\tau)$ . The amplitude of this retarded correlation ( $2\bar{\mathcal{K}}(0) = 2$ ) is however only 10% of the observed correlation ( $\mathcal{L}_I(0) = 18$ ). We have fitted the observed correlation for indices by a sum of two exponentials, with only the parameters of the ‘fast’ one left free, the slow one being fixed by fitting individual stocks. The resulting fit (not shown) was not significantly better (nor worse) than the single exponential fit of Fig. 2. Given the amount of noise in the data, it is difficult to prove or disprove the presence of

the slowly decaying correlation. Nevertheless, we argue that it should be present for reasons of consistency between the index and its constituents.

Let us finally estimate the contribution of  $\gamma(\tau)$  to the individual stock leverage effect. A simple computation gives, to lowest order:

$$\mathcal{L}_i(\tau) = -2\bar{\mathcal{K}}(\tau) - \beta_i^3 \left(\frac{\sigma_I}{\sigma_i}\right)^4 \gamma(\tau). \quad (23)$$

Since the market volatility  $\sigma_I$  is a factor 3 smaller than the volatility of individual stocks  $\sigma_i$  [25], the prefactor in front of  $\gamma(\tau)$  is of order 1/100. Therefore, even if  $\gamma(\tau)$  is ten times larger than  $\bar{\mathcal{K}}(\tau)$ , the influence of the market leverage effect on individual stocks is effectively suppressed due to relatively large ratio between the stock volatility and the market volatility. Again, due to the amount of noise in the data, it is difficult to confirm directly the presence of the  $\gamma(\tau)$  contribution in  $\mathcal{L}_i(\tau)$ . However, since this contribution is small and decays relatively fast, we believe that its role for individual stocks can safely be neglected.

## 5 Conclusion

In this paper, we have investigated quantitatively the so-called leverage effect, which corresponds to a negative correlation between future volatility and past returns. We have found that this correlation is moderate and decays over a few months for individual stocks, and much stronger but decaying much faster for stock indices. In the case of individual stocks, we have found that the magnitude of this correlation can be rationalized in terms of a retarded effect, which assumes that the reference price used to set the scale for price updates is *not* the instantaneous price but rather a moving average of the price over the last few months. This interpretation, supported by the data on U.S., European and Japanese stocks, appears to us rather likely, and defines an interesting class of stochastic processes, intermediate between purely additive (valid on short time scales) and purely multiplicative (relevant for long time scales), first advocated in [7]. For stock indices, however, this interpretation breaks down and a specific market panic phenomenon seems to be responsible for the enhanced observed negative correlation between volatility and returns (and in turn to the strong skews observed on index option smiles). Interestingly, this effect appears to decay rather quickly, over a few days. As a simple one factor model shows, the two effects (retardation and panic) are not incompatible and should both be present in individual stocks and stock indices. However the relative amplitude of the retardation effect for indices and of the panic effect for stocks are both too small to be unambiguously detected with only stock or index data. Note that in both cases, the correlation between volatility and returns appears to decay exponentially, in strong contrast to volatility-volatility correlations which decay as a power-law. This power-law behavior has recently lead to the construction

of a beautiful multifractal model [9]. It would be interesting, from a theoretical point of view, to generalize this model to account for a scale invariant leverage effect. Work in this direction is underway.

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## Appendix

The exponential retarded model can be reformulated in terms of a product of random  $2 \times 2$  matrices. Define the vector  $\vec{V}(t)$  as the set  $S_R(t), S(t)$ . One then has:

$$\vec{V}(t+1) = \mathcal{M}(t)\vec{V}(t) \quad (24)$$

where  $\mathcal{M}(t)$  is a random matrix whose elements are  $\mathcal{M}_{11}(t) = \alpha$ ,  $\mathcal{M}_{12}(t) = 1 - \alpha$ ,  $\mathcal{M}_{21}(t) = \sigma\epsilon(t)$  and  $\mathcal{M}_{22}(t) = 0$ . Using standard results on products of random matrices, one directly finds that the large time statistics of both components of  $\vec{V}$  is log-normal.

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- [23] One can also consider the case where  $\sigma_i$  is a random, time dependent quantity, to account for volatility clustering.

- [24] We have noted however that the very first point  $\tau = 1$  day is systematically, and significantly, above the value  $-2$ . We have not found a simple interpretation of this fact.
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